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Domenico Delli Gatti, Filippo Gusella, Giorgio Ricchiuti

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Endogenous vs Exogenous Instability: An Out-of-Sample Comparison

Domenico Delli Gatti
CLE, Università Cattolica del Sacro Cuore, Milano
Filippo Gusella
Università degli Studi di Firenze
CLE, Università Cattolica del Sacro Cuore, Milano
Giorgio Ricchiuti
Università degli Studi di Firenze
CLE, Università Cattolica del Sacro Cuore, Milano

Abstract

Given the unobserved nature of expectations, this paper employs latent variable analysis to examine three financial instability models and assess their out-of-sample forecasting accuracy. We compare a benchmark linear random walk model, which implies exogenous instability phenomena, with a linear state-space model and a nonlinear Markov regime-switching model, both of which postulate endogenous fluctuations phenomena due to heterogeneous behavioral heuristics. Using the S&P 500 dataset from 1990 to 2019, results confirm complex endogenous dynamics and suggest that the inclusion of behavioral nonlinearities improves the model's predictability both in the short, medium, and long run.

Keywords: Endogenous instability, exogenous instability, behavioral model, forecasting analysis

Jel codes: C13, C51, E37, G01, G10

Corrisponding Author: Domenico Delli Gatti. domenico.delligatti@unicatt.it, via Necchi 5, ufficio 303, Milano.

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1 Introduction

Financial instability has always been at the center of economic studies and the subject has been analyzed in great detail in macroeconomic literature (Kindleberger, 1978; Minksy, 1985). Despite the absence of a consensus on the causes and nature of financial instability, it is possible to identify two schools of thought that have emerged over time. The first one is based on the *efficient market theory* (Fama, 1991). According to this view, instability can be seen as a consequence of an exogenous shock to the system in the context of a representative economic agent with perfect information and rational expectations (Muth, 1961). In contrast, for the *behavioral theory* (Beja and Goldman, 1980; Schleifer and Summers, 1990; Shiller, 2003; Thaler, 2015; Vikash et al., 2015), instability can be approached as an endogenous phenomenon in the system. In the absence of perfect information about the system, instability can emerge from the out-of-equilibrium interaction of boundedly heterogeneous agents. Pioneered by Day and Huang (1990), Chiarella (1992), De Grauwe et al. (1993), Lux (1995), and Brock and Hommes (1998), according to such studies, the behavioral heterogeneity of interacting investors involves agents' expectations formation. This gives rise to what is called Heterogeneous Agent Models (HAMs). For comprehensive reviews of such models, see Hommes (2021).

Over the last two decades, different empirical methodologies have been employed within the HAM literature to investigate the nature of expectations and their potential effects on the financial and economic spheres (for recent surveys, refer to Lux and Zwinkels, 2018 and Ter Ellen and Verschoor, 2018). Such empirical studies do not arise in opposition to each other, but they are complementary. For instance, a strand of research focuses on the possibility of extracting different expectation formations based on behavioral attitudes from survey evidence (Franke and Westerhoff, 2017). At the same time, laboratory financial market experiments go precisely in the same direction, confirming how the single representative rational hypothesis can be substituted with the idea that heterogeneous agents interact following simple heuristic forecasting rules (Colasante et al., 2018; 2019; 2020; 2020; Bao et al., 2020; 2021). Along the same line, direct likelihood or indirect simulated estimation methods can empirically identify different agent groups (Franke and Westerhoff, 2011; Kukacka and Barunik, 2017; Frijns and Zwinkels, 2018; Chen and Lux, 2018; Ter Ellen et al., 2021; Lux, 2021a; Schmitt, 2021). These works indicate that different types of agents can populate the market, for example, those who follow destabilizing trading rules (chartists) and those who follow stabilizing trading strategies (fundamentalists).

Despite the different aforementioned approaches, a central tenet remains widely accepted: the expectation heterogeneity amongst distinct agent groups. These different behavioral heuristics represent latent factors or hidden market regimes that are not directly observable but are believed to play a crucial role in shaping the observed financial data. Incorporating such components as unobserved state components allows researchers to understand better the potential impact of behavioral factors on financial markets. Vigfusson (1997) was the first one to employ the Markov regime-switching methodology within the foreign exchange market, considering heuristic behaviors as distinct states. This approach was subsequently adopted by other researchers in spot and forward exchange rate market (Ahrens and Reitz, 2003; Li et al. 2013), the stock market (Chiarella et al., 2012) and the housing market (Chia et al., 2017). Recently, continuous state-space Markov processes, or simply state-space models, have been introduced. This approach was initiated by Thomas Lux (2018b; 2021b), who advocated for state-space representation and has been pursued by other scholars, such as Gusella and Stockhammer (2021), Gusella and Ricchiuti (2022), and Gusella (2022).

As the latent factors can be associated to different beliefs among the agents, the methods mentioned above become fundamental to empirically investigate whether these components can generate endogenous dynamical instability in price formation. Indeed, it is possible to pass from a structural HAM to a reduced-form linear or non-linear econometric model with latent states so as to investigate potential cyclical dynamics as a consequence of agents' beliefs. At the same time, an out-of-sample comparison would help us to understand the power predictability of different visions of instability. On this point, we are not aware of any empirical comparison between the linear state-space model and the nonlinear switching Markov process of the chartist/fundamentalist, both of which postulate endogenous instability, with the benchmark random walk hypothesis. In performing it, we can understand whether the endogenous component is superior to the exogenous component in explaining instability phenomena and if highlighting nonlinear local instabilities enhances the model's explanation compared to linear global instability. This is where our paper comes in. To perform it, the analysis is divided into different parts. In the following, we provide the main steps of our analysis.

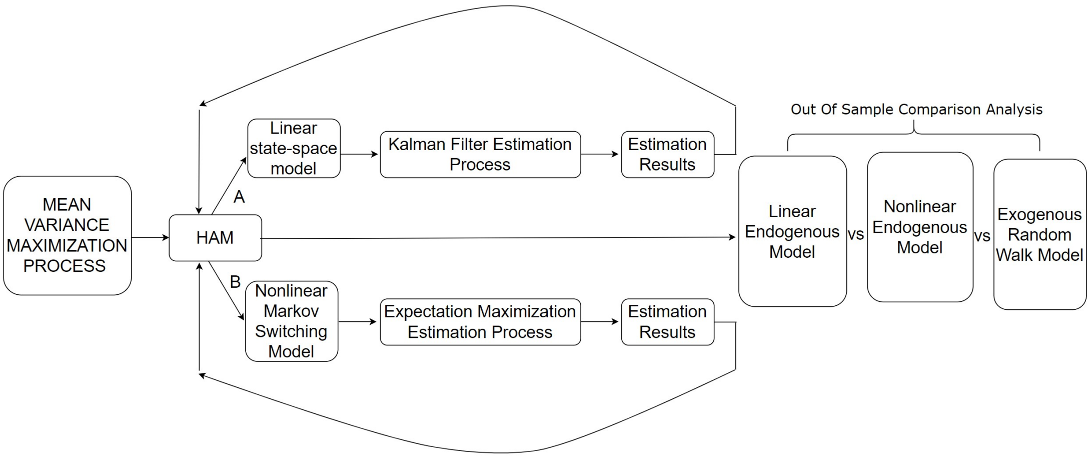

Figure 1: Methodology.

We assume that the economy is populated by mean-variance utility optimizer investors (see Fig. 1 for a schematization). Based on this, we construct a conventional heterogeneous agent-based model wherein agents are classified into two groups characterized by distinct expectation formation methods: the chartist and the fundamentalist groups.[1] By doing that, we end up with a 2-type heterogeneous agent model into a standard small-scale dynamic asset pricing framework. Because beliefs of agents are latent variables, we first convert the structural theoretical model into a reduced linear state-space form (line A). In the state-space model, we distinguish between observed and unobserved variables. In our case, the asset price will be considered the observed variable. Conversely, the beliefs of the agents will be the unobserved components that dynamically influence the observation value over time. In this way, we can establish possible endogeneity that comes from the heuristics of the agents. In fact, we are able to study the mathematical conditions to obtain complex endogenous dynamics. Compatible with previous research, analytical results confirm that these conditions depend on the parameter associated with the group of trend followers. In contrast to Lux (2018), our approach involves fixed quotas, and we treat the agents' strategies as unobservable. This enables us to uncover hidden heuristics and grasp their dynamic impact on the observable asset price, with a specific emphasis on analyzing cycles. Unlike Gusella and Stockhammer (2021) and Gusella (2022), additional heuristics characterizing agent behavior are introduced into the model, specifying the fundamental asset price as an exogenous value.

Secondly, to overcome the fixed linearity assumption on the percentage of agents in the market (Gusella and Ricchiuti, 2022), a nonlinear two-state Markov switching model will be

[1] Throughout the remainder of the paper, the terms chartists, trend followers, and technical traders are used interchangeably.

considered (line B). Indeed, investors' strategies are likely to change over different time periods, reflecting intervals where a strategy prevails over the other. If the state of fundamentalists dominates, then the price dynamics should be tied to the price dynamics of the fundamental price. If the state of trend followers dominates, then the price dynamics should be linked to its previous values. The probability of being in one of the two regimes approximates the weight assigned to the correspondent rule. In fact, although various groups of agents adopt different expectation formations, the auctioneer determines the market-clearing price based on a weighted average of the two forecasts. This decision-making process is directly informed by the probability associated to the latent state variables at each period. Unlike Vigfusson (1997), Ahrens and Reitz (2003), Li et al. (2013) and Chia et al. (2017) we incorporate regime-switching models into equity prices, as done by Chiarella et al. (2012). Unlike Chiarella et al. (2012), the switching process involves regime changes for both speculative strategies, not just within a single group.

For both the two specifications, Monte Carlo maximum likelihood estimation analysis is performed. In the first case, with the use of the Kalman filter algorithm to recover the log-likelihood function and the parameters associated to the beliefs of the agents. In the second case, we use the expectation-maximization algorithm to optimize the data likelihood. Finally, we contribute to the existing literature on latent state analysis by comparing the linear (c&f) model, the nonlinear (c&f) switching model, and the benchmark random walk model with each other. In order to do so, we compare the three specifications in a rolling window out-of-sample forecasting analysis to see which type of specification increases the predictability of the model.

Analyzing the S&P 500 data on a time window ranging from 1990 to 2019 at monthly frequency, we show evidence of endogenous cycles. Furthermore, covering an out-of-sample evaluation period from January 2017 to December 2019, this study suggests that including behavioral nonlinearity improves the model's predictability in the short, medium and long run over the linear behavioral model and the benchmark random walk hypothesis.

The rest of the paper is organized along the following lines. In section 2, we set the heterogeneous agent model (2.1). The model is firstly converted into a linear state-space form (section 2.2), from which the conditions for endogenous cycles can be mathematically derived. Secondly, the extension from a linear model to a nonlinear Markov approach is implemented (section 2.3). In section 3, Monte Carlo estimation results are presented. In section 4, the rolling-window out-of-sample forecasting analysis is performed to compare the three specifications. Section 5 concludes.

2 Methodology

2.1 General set-up

In line with Brock and Hommes (1997, 1998) and Hommes (2005), financial operators at time t can invest in two asset types - a risk-free asset offering a fixed rate of return r and a risky asset with price p and a stochastic dividend y. The dynamics of wealth at the end of period $t+1$ can be captured by the following dynamic equation:

$$W_{t+1} = (1+r)W_t + \left[p_{t+1} + y_{t+1} - (1+r)p_t\right] z_t,$$

where W indicates the end-of-period wealth while z is the number of shares of the risky asset purchased at date t. Agents are assumed to be mean-variance maximizers, such that:

$$\max_{zt} \left\{ E_t^i \left[W_{t+1}\right] - \frac{a}{2}\sigma^2 \right\},$$

where E_t^i is the expectation operator of agents of type i, σ^2 is the conditional variance of the wealth, supposed to be constant over time and equal for all types of agents, and $a > 0$ represents the risk aversion parameter. Maximizing with respect the demand z_t^i by trader type i, we obtain:

$$E_t^i\left(p_{t+1}\right) + E_t^i\left(y_{t+1}\right) - (1+r)p_t - z_t^i a\sigma^2 = 0,$$

from which:

$$z_t^i = \frac{E_t^i\left(p_{t+1}\right) + E_t^i\left(y_{t+1}\right) - (1+r)p_t}{a\sigma^2}.$$

Without loss of generality, expectations regarding dividends are uniform across all trader types and equal to the conditional expectation:

$$E_t^i\left(y_{t+1}\right) = \bar{y},$$

such that:

$$z_t^i = \frac{E_t^i\left(p_{t+1}\right) + \bar{y} - (1+r)p_t}{a\sigma^2}.$$

Suppose now there are I groups of investors, each with diverse expectations regarding the future price p_{t+1}. We assume that δ^i is the fraction (market share) of type i investors, with $\sum_{i=1}^{I} \delta_i = 1$. Total demand for the risky asset is then given by:

$$\sum_i^I \delta^i \frac{E_t^i\left(p_{t+1}\right) + \bar{y} - (1+r)p_t}{a\sigma^2}.$$

The supply of the risky asset per investor is equal to zero, such that, from the equilibrium condition:

$$\sum_i^I \delta^i \frac{E_t^i\left(p_{t+1}\right) + \bar{y} - (1+r)p_t}{a\sigma^2} = 0,$$

we get:

$$(1+r)p_t = \sum_i^I \delta^i E_t^i\left(p_{t+1}\right) + \bar{y}.$$

Consistent with HAMs, the heterogeneity of agents is attributed to their distinct methods of expectation formation. Specifically, financial traders employ either a fundamental or a technical expectation rule to project future prices. Thereby:

$$(1+r)p_t = \delta E_t^f\left(p_{t+1}\right) + (1-\delta)\, E_t^c\left(p_{t+1}\right) + \bar{y}.$$

where E_t^f and E_t^c are the expectation operators of fundamentalists and chartists, respectively.

Under the assumption that both the risk-free rate and the constant dividend are equal to zero, the price of the risky asset is expressed as follows:

$$p_t = \delta E_t^f\left(p_{t+1}\right) + (1-\delta)\, E_t^c\left(p_{t+1}\right). \tag{1}$$

We now formalize the expectation formation of the two groups of agents considered. Fundamentalists believe in the efficient market theory, expecting the price to be equal to the fundamental value p_t^f:

$$E_t^f\left(p_{t+1}\right) = p_t^f. \tag{2}$$

Conversely, chartists speculate considering the price at the previous time:

$$E_t^c\left(p_{t+1}\right) = p_t^f + \beta\left(p_{t-1} - p_{t-2}\right), \qquad with \quad \beta > 0 \tag{3}$$

where parameter β represents the reaction coefficient, indicating the degree to which speculators extrapolate past changes in the asset market.

Substituting Eqs. 2 and 3 in Eq. 1, we obtain:

$$p_t = \delta\left(p_t^f\right) + (1-\delta)\left(p_t^f + \beta\left(p_{t-1} - p_{t-2}\right)\right),$$

from which:

$$p_t = p_t^f + (1-\delta)\,\beta\left(p_{t-1} - p_{t-2}\right).$$

With respect to the belief of chartists, this last equation can be rewritten in the following way:

$$p_t = p_t^f + (1-\delta)\,B_t^c, \tag{4}$$

with:

$$B_t^c = \beta\left(p_{t-1} - p_{t-2}\right).$$

Finally, for the construction of the fundamental value, we derive it from the Gordon growth model (Gordon, 1959) using S&P500 data on dividends. Defining $\tilde{g}$ as the average growth rate of dividends, $\tilde{r}$ the average required return, and d_t the dividend flow, the fundamental value of asset price can be defined as:

$$p_t^f = d_t\frac{(1+\tilde{g})}{(\tilde{r}-\tilde{g})}.$$

Following Chiarella et al. (2012), we assume that $\tilde{r}$ is equal to the sum of the average dividend yield $\tilde{y}$ and the average rate of capital gain $\tilde{x}$. The Gordon growth model implies that $\tilde{x}$ is equal to $\tilde{g}$, so as to obtain:

$$p_t^f = d_t\frac{(1+\tilde{g})}{\tilde{y}}.$$

2.2 Linear state-space form

To convert the structural theoretical model of section 2.1 in a reduced state-space form, Eq. 4 is substituted in the belief function of chartists so as to obtain:

$$B_t^c = \beta\left(p_{t-1}^f + (1-\delta)\,B_{t-1}^c - p_{t-2}^f - (1-\delta)\,B_{t-2}^c\right).$$

In line with Franke (2008) and Lux (2021a), the fundamental value is assumed to follow a Brownian motion with increments $\varepsilon_t \sim \mathrm{N}\left(0, \sigma_\varepsilon\right)$. This hypothesis is tested on the Gordon fundamental time series. The obtained results of the Augmented Dicky Fuller test fail to reject the null hypothesis of a unit root.[2] We thus have:

$$B_t^c = \beta\,(1-\delta)\,B_{t-1}^c - \beta\,(1-\delta)\,B_{t-2}^c + \varepsilon_t.$$

In a matrix-vector formulation, the state-space form assumes the following form:

[2]The results are not included in this part of the paper due to space limitations. Results are available upon request.

$$p_t = p_t^f + \begin{bmatrix} (1-\delta) & 0 \end{bmatrix} \begin{bmatrix} B_t^c \\ B_{t-1}^c \end{bmatrix} \tag{5}$$

$$\begin{bmatrix} B_t^c \\ B_{t-1}^c \end{bmatrix} = \begin{pmatrix} \beta\,(1-\delta) & -\beta\,(1-\delta) \\ 1 & 0 \end{pmatrix} \begin{bmatrix} B_{t-1}^c \\ B_{t-2}^c \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ 0 \end{bmatrix}. \tag{6}$$

The measurement equation is represented by Eq. 5, whereas the unobserved state equation, containing the belief function of chartists, is depicted by Eq. 6. As formalized in our model, the belief function impacts price dynamics, thereby potentially inducing endogenous instability in the form of fluctuations. The dynamics, fully expressed by the eigenvalues associated with the transition matrix, can be studied by solving for the determinant of the characteristic equation:

$$\det \begin{pmatrix} \beta\,(1-\delta) - \lambda & -\beta\,(1-\delta) \\ 1 & -\lambda \end{pmatrix} = 0.$$

Once the determinant of the characteristic equation is obtained, the condition to obtain endogenous fluctuations is reflected in complex eigenvalues if the following condition on the discriminant is satisfied:

$$\Delta = \beta\,(1-\delta)\,[\beta\,(1-\delta) - 4] < 0,$$

i.e., as a sufficient and necessary condition:

$$0 < \beta\,(1-\delta) < 4. \tag{7}$$

From the last equation, it is crucial to have a certain percentage of chartists in the market and a positive reaction parameter. If either β or $1-\delta$ are equal to zero, the market price will reflect the fundamental price, aligning with the efficient market hypothesis.

If condition in Eq. 7 holds, the eigenvalues assume the following form:

$$\lambda_{1,2} = \frac{\beta\,(1-\delta)}{2} \pm i\frac{\sqrt{-\Delta}}{2} = a \pm ib,$$

or in an equivalent trigonometric form:

$$\lambda_{1,2} = \rho\,(\cos\theta \pm \sin\theta) \quad with \quad \rho = \left(a^2 + b^2\right)^{1/2}.$$

If complex dynamics is satisfied and $\rho = \sqrt{\beta\,(1-\delta)} = 1$, we obtain constant endogenous fluctuations. If $\rho = \sqrt{\beta\,(1-\delta)} < 1$, we observe damped endogenous fluctuations, while explosive endogenous fluctuation if $\rho = \sqrt{\beta\,(1-\delta)} > 1$.

To simplify notation, we define a_{11} as $\beta\,(1-\delta)$ and a_{12} as $-\beta\,(1-\delta)$. Thus, the state-space model takes the following form:

$$p_t = p_t^f + HB_t$$

$$B_t = AB_{t-1} + \varphi_t \qquad \varphi_t \sim \mathrm{N}\,(0, Q)\,.$$

where p_t is the observable asset price,

$$B_t = \begin{bmatrix} B_t^c \\ B_{t-1}^c \end{bmatrix}$$

is the state vector,

$$H = \begin{bmatrix} (1-\delta) & 0 \end{bmatrix}$$

is the measurement matrix,

$$A = \begin{pmatrix} a_{11} & a_{12} \\ 1 & 0 \end{pmatrix}, \quad with \quad \begin{cases} a_{11} = (1-\delta)\,\beta \\ a_{12} = -\,(1-\delta)\,\beta \end{cases} \tag{8}$$

is the transition matrix and φ_t is the vector containing the state disturbance of the unobserved component, normally distributed with mean zero and variances collected in the diagonal matrix Q.

2.3 Nonlinear Markov regime-switching form

The linear formalization of the previous section has a potential shortcoming. In fact, agents can switch between the two strategies during different phase of time. To capture this feature, the previous framework is enriched with a nonlinear two-states Markov switching mechanism.

Since we consider two groups of agents with different expectations that influence price dynamics, we can consider the system in two different regimes (or states): the speculative and the fundamentalist regime. Each regime forms two submodels. In one regime, the fundamentalists dominate the market and the dynamics of the price should be reflected by the variation of fundamental price. In the second regime, speculative strategies dominate the market and the unobserved submodel is driven by an autoregressive process. The switching mechanism is controlled by an unobservable state variable that follows a first-order Markov chain. Thus, a specific behavioral rule (for example, the fundamentalists' behavior) only prevails for a specific time, after which it can possible "switch" to another behavioral rule (the chartists' behavior).

Empirically, to study the dynamic behavior of asset price in each regime, we implement a Markov-switching dynamic regression (MSDR) model (Hamilton, 1994; 2016). With respect to our model, to improve the maximization of the log-likelihood function, the model is expressed in differences rather than levels. Considering the first difference for asset price leads to the following equation:

$$p_t - p_{t-1} = \left(p_t^f - p_{t-1}^f\right) + (1-\delta)\beta\,(p_{t-1} - p_{t-2}) - (1-\delta)\beta\,(p_{t-2} - p_{t-3})\,.$$

.

Such that, with $a_{11} = (1-\delta)\beta$, $a_{12} = -(1-\delta)\beta$ and the Brownian motion for the fundamental value, we obtain:

$$p_t - p_{t-1} = \varepsilon_t + a_{11}\,(p_{t-1} - p_{t-2}) + a_{12}\,(p_{t-2} + p_{t-3})\,,$$

from which:

$$p_t - p_{t-1} = \varepsilon_t\,(s_t) + \left[a_{11}\,(p_{t-1} - p_{t-2}) + a_{12}\,(p_{t-2} + p_{t-3})\right](s_t)\,.$$

with:

$$p_t - p_{t-1} = \begin{cases} fundamentalist \quad regime & if\ s_t = f \\ chartists \quad regime & if\ s_t = c \end{cases}$$

where s_t is the latent state-space discrete-time Markov chain representing the switching mechanism among the two regimes.

There are four kinds of state transitions possible between the two states:

- From state f to state f: This transition happens with probability $p_{ff} = P\left(s_t = f\,|s_{t-1} = f\right)$[3]
- From state f to state c: This transition happens with probability $p_{fc} = P\left(s_t = c\,|s_{t-1} = f\right)$

[3]This is read as the probability that the system is in regime 0 (fundamentalist regime) at time t, given that it was in the same regime at the previous time $(t-1)$.

- From state c to state f: This transition happens with probability $p_{cf} = P\left(s_t = f \,|s_{t-1} = c\right)$
- From state c to state c: This transition happens with probability $p_{cc} = P\left(s_t = c \,|s_{t-1} = c\right)$

with:

$$p_{ff} + p_{fc} = 1 \quad and \quad p_{cf} + p_{cc} = 1.$$

In the end, s_t depends on s_{t-1} according to the following state transition matrix:

$$\begin{bmatrix} p_{ff} & p_{fc} \\ p_{cf} & p_{cc} \end{bmatrix}.$$

This extension, with the filtering process of the unobserved latent states, offers valuable insights into how different market strategies prevail during crucial periods, helping us to better understand the local dynamics of financial markets.

3 In sample estimation results

Once the structural model has been transformed into reduced form for the estimation process, we estimate both the linear and nonlinear version with the S&P 500 data using the closing adjusted values from January 1990 to December 2019 at monthly frequency.[4] For the econometric analysis, all the series are transformed in log levels.

For the linear model, we utilize the recursive Kalman filter algorithm to determine the optimal estimator for the state variable and to estimate the parameters via the maximum likelihood function (see Durbin and Koopman, 2012; Enders, 2016). This prediction error decomposition process involves iterative computation based on one-step prediction and updating equations over the state-space form to estimate a_{11} and a_{12} as well as the percentage of chartists in the market $(1 - \delta)$. Based on 1000 Monte Carlo simulations, we verify whether the condition for endogenous fluctuation phenomena holds $\{\beta\,(1-\delta)\,[\beta\,(1-\delta) - 4] < 0\}$. Throughout the estimation process, we assume the percentage of chartists in the market between zero and one. Additionally, we enforce the equality constraint $a_{11} + a_{12} = \beta\,(1-\delta) - \beta\,(1-\delta) = 0$ to ensure a unique positive reaction coefficient β retrieved from system 8.

Differently from the linear case, Markov switching estimation is performed using the Expectation Maximization algorithm following the method developed for time series analysis by Hamilton (1994, 2016). The EM iteration involves an expectation (E) step and a maximization (M) step. During the expectation step, the algorithm computes the expected values of latent variables given the current parameter estimates. Subsequently, the maximization step computes parameter values that maximize the anticipated log-likelihood obtained from the E step. These estimated parameters are then employed to recover the distribution of latent variables in the subsequent E step. As before, estimates are based on 1000 Monte Carlo randomly generated inputs.

The empirical results are divided into two main subsection. In the first, we present results from the linear state-space model. In the second subsection, we present the results of the nonlinear Markov model.

3.1 Linear behavioral model

Table 1 presents Monte Carlo results obtained from estimating the model in a linear state-space form. The table is organized in 3 blocks. The uppermost part displays coefficients for the

[4]We have considered data up to the year 2019 in our analysis to avoid the possible confounding effects of the COVID-19 period.

cyclical parameters, followed by the values of the percentage and reaction coefficient of chartists in the second row, along with the associated value for the state disturbance in the third row. The middle block provides information about the endogenous cycles. The bottom part of the table includes general estimation information.

We focus on the cyclical parameters of the transition matrix. Our findings from the linear state-space model suggest that cyclical conditions are respected in the necessary and sufficient conditions $\{\beta(1-\delta)[\beta(1-\delta)-4]<0\}$. In particular, the coefficient $a_{12} = 0.5398$ and $a_{21} = -0.5398$ are statistically significant at one percent, confirming the nature of complex eigenvalues. The existence of complex eigenvalues underscores how price dynamics reflects endogenous fluctuation phenomena, characterized by dampened fluctuations with a modulus equal to 0.73 $[\sqrt{\beta(1-\delta)}<1]$.

The estimated percentage of trend followers in the market is $1-\delta = 0.31$, indicating that they are in the minority compared to agents who follow fundamentalist behavior ($\delta = 0.69$) with a reaction coefficient of $\beta = 1.74$.

To summarize, our results align with HAMs tradition, which documents the presence of different agents in the market with different beliefs. In addition, with the state-space analysis, we document the crucial effect of heuristics in the determination of price dynamics, thereby supporting the hypothesis that endogenous cyclical conditions can arise from the speculative positions taken by agents and formalized in our theoretical model.

Table 1: Monte Carlo Results [State-Space Model]

	Cyclical Parameters		
	a_{11}	a_{12}	
Values	0.5398*** [0.5375, 0.5421]	-0.5398*** [-0.5421, -0.5375]	
	Percentage and Reaction Coefficients		
	δ	$1-\delta$	β
Values	0.69	0.31*** [0.3099, 0.3164]	1.74
	State Disturbance		
	σ_ε		
Values	1.0238*** [0.9678, 1.0797]		
	Cyclical Conditions		
$\beta(1-\delta)[\beta(1-\delta)-4]<0$		*Fulfilled*	
$(a^2+b^2)^{1/2}$		*Damped*	
	Info Model		
Eigenvalues	$-0.2699 \pm 0.6833i$		
Modulus	0.73		

Notes: Confidence interval in squared brackets.
*, **, *** denotes statistical significance at the 10%, 5%, and 1% levels respectively.

3.2 Nonlinear behavioral model

We now present the results obtained from the nonlinear specification. Table 2 is divided into three main blocks. Similarly to the previous table, at the top of the table, the first row displays coefficients for the cyclical parameters. The second row indicates the estimated transition state probabilities denoted as p_{00}, p_{01}, p_{10}, and p_{11}, providing information about the probabilities of remaining in the same state (speculative or fundamentalist) or transitioning between the two states. In the middle, we provide information about the endogenous cycles. Finally, the bottom section summarizes the general estimation information.

As can be seen in Table 2, the coefficients $a_{11} = 0.04$ and $a_{21} = -0.04$, associated with the trend follower strategy, confirm the presence of complex eigenvalues. Complex eigenvalues in the system indicates that endogenous fluctuation phenomena characterize the price dynamics. At the same time, these fluctuations exhibit a damped behavior, with a modulus equal to 0.46.

Of particular interest is the estimated transition state probability. The values in the table are as follows: $p_{00} = 0.9425$, $p_{01} = 0.0575$, $p_{10} = 0.0833$, and $p_{11} = 0.9167$. These results indicate a high probability of remaining in the same state (0.9425 for fundamentalists and 0.9167 for trend followers), with a lower probability of transitioning between the two states (0.0575 from fundamentalists to chartists and 0.0833 in the opposite direction) (see Fig. 2). This finding is consistent with previous studies, which have demonstrated a high degree of persistent behavioral attitudes among traders (Anufriev and Hommes, 2012). Furthermore, through the application of filtering and smoothing techniques to uncover the latent state dynamics, we approximate the weight of the behavioral strategies, observing their degree of manifestation within specific years of interest. Figures 3 and 4 illustrate that trend followers dominated during the dot.com crisis and the global financial crisis, while the fundamentalist strategy prevailed in the years following the burst of the bubbles, guiding market dynamics in the subsequent period.

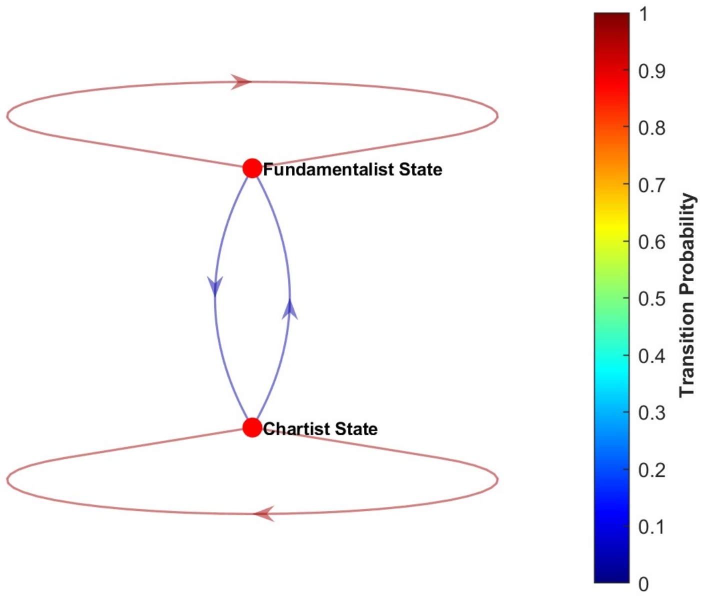

Figure 2: Structure and evolution of the Markov chain with the probability of transition.

Table 2: Monte Carlo Results [Nonlinear Switching Model]

	Cyclical Parameters			
	a_{11}	a_{12}		
Values	0.04***	-0.04***		
	[0.03, 0.05]	[-0.05, -0.03]		
	Estimated Transition State Probability			
	p_{00}	p_{01}	p_{10}	p_{11}
Values	0.9425	0.0575	0.0833	0.9167
	State Disturbance			
	σ_{ε}			
Values	0.0018***			
	[0.0017, 0.0019]			
	Cyclical Conditions			
$a_{11}^2 - 4a_{12} < 0$	*Fulfilled*			
$(a^2 + b^2)^{1/2}$	*Damped*			
	Info Model			
Eigenvalues	-0.0200 ± 0.1990i			
Modulus	0.46			

Notes: Confidence interval in squared brackets.

*, **, *** denotes statistical significance at the 10%, 5%, and 1% levels respectively.

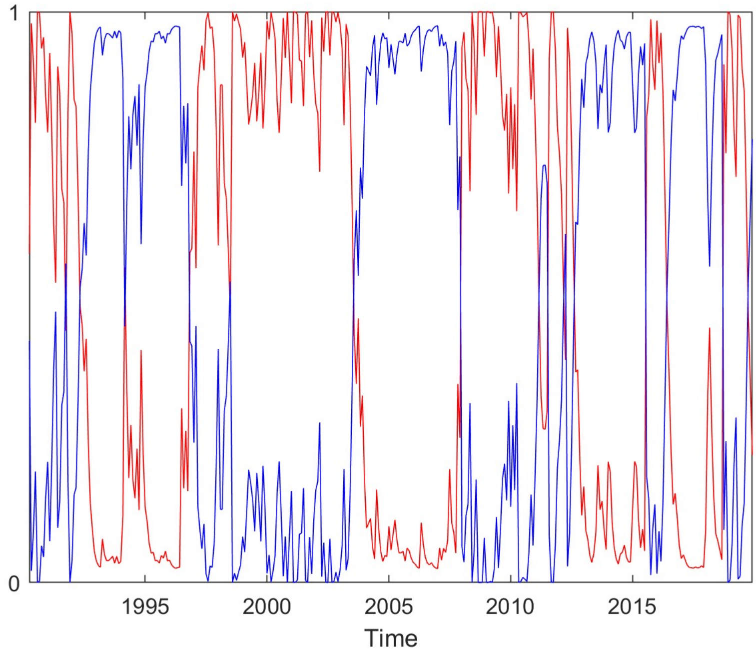

Figure 3: Filtered Unobserved Chartist State with Estimated State Probability in red. Filtered Unobserved Fundamentalists State with Estimated State Probability in blue.

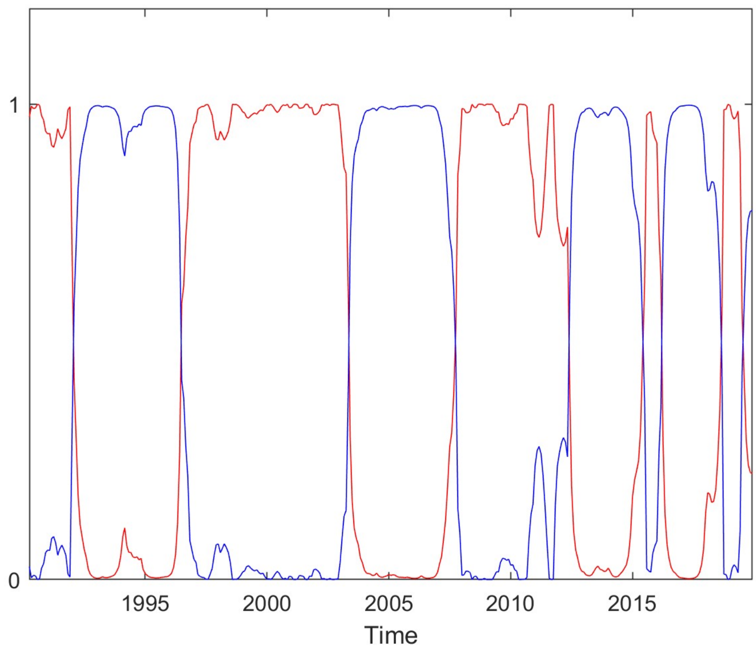

Figure 4: Smoothed Unobserved Chartist State with Estimated State Probability in red. Smoothed Unobserved Fundamentalists State with Estimated State Probability in blue.

4 Out-of-sample comparison

After estimating the models, we proceed to compare the two, along with the benchmark random walk model, through an out-of-sample state-space analysis. We divide our data sample between Jan 1990 - Dec 2016 (in-sample period) and Jan 2017 - Dec 2019 (out-of-sample) so as to ensure many forecast observations to conduct inference. Concerning the linear forecasting procedure, the prediction error decomposition approach is repeated for all the out-of-sample periods in the forecasting exercise (Harvey, 2006). For the nonlinear version, we rely on the one-step-ahead point forecast for a Markov-switching dynamic regression model developed by Hamilton (1990).

A rolling window forecasting analysis is performed for different forecasting horizons $h = (1, 2, 3, ...12)$. This is important for three main reasons. First, since the sample separation is somewhat arbitrary, we avoid forecasting results sensitive to the sample selection. Second, we aim to mimic the behavior of financial traders who update their forecasting when new information arrives in each period. Finally, in this way, we can distinguish between short-period forecasting horizons $h = (1, 2, 3, 4)$, medium-period forecasting horizons $h = (5, 6, 7, 8)$ and long-period forecasting horizons $h = (9, 10, 11, 12)$ so as to check how the three specifications perform.

Proceeding in this way, the forecasting recursion involves a rolling estimation window of 325 observations with a fixed sample size. We estimate the model using the initial 325 observations $(p_1, p_2, ..., p_{325})$, subsequently generating forecasts for the first $h = 1, 2, 3, ...12$ months ahead $(p_{325+1}, p_{325+2}, ..., p_{325+12})$. Likewise, we derive the subsequent h-month-ahead forecasts $(p_{326+1}, p_{326+2}, ..., p_{326+12})$ by utilizing the data sequence $p_2, p_3, ..., p_{326}$. This iterative procedure continues until we generate the final h-month-ahead forecasts $(p_{348+1}, p_{348+2}, ..., p_{348+12})$ based on the observations $p_{24}, p_{25}, ..., p_{348}$. This recursion leads to a sequence $k_h = 24$ for each forecasting horizon. See Fig. 5 for a graphical explanation of the rolling window forecasting analysis.

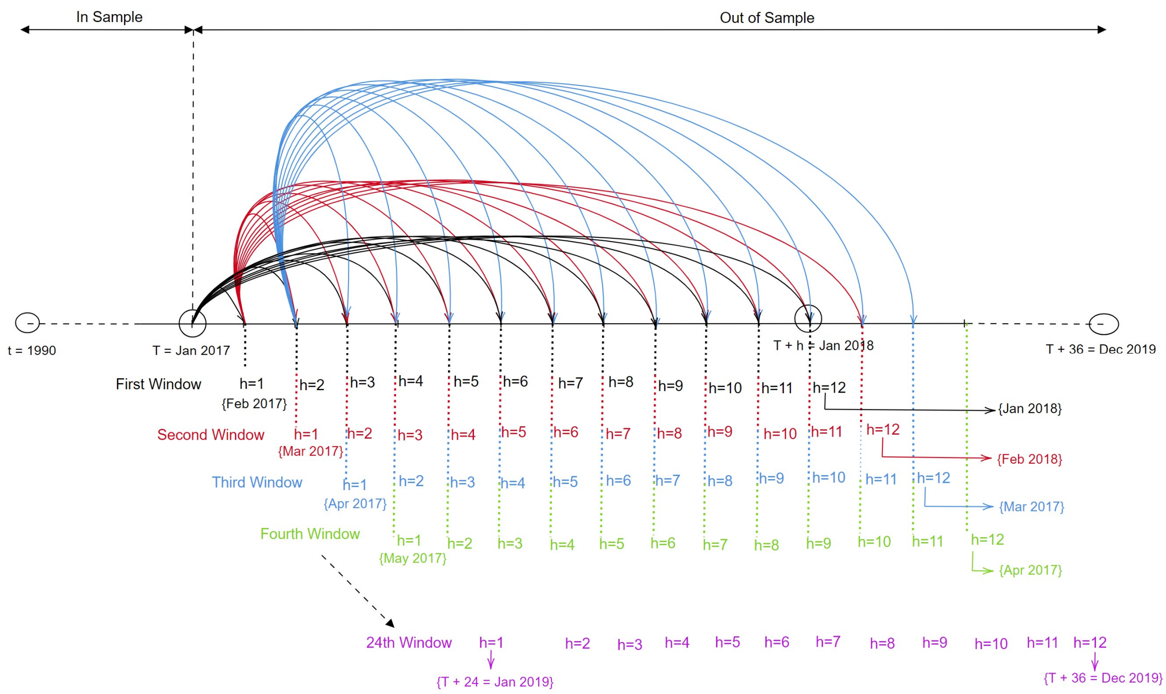

Figure 5: Rolling Window Forecasting Procedure.

Once the forecasting procedure has been implemented, the root mean square error (RMSE) and the mean absolute error (MAE) are calculated to compare the forecasting results for the different forecasting horizons. At the same time, we perform the Diebold-Mariano test to compare the predictive performance of different forecasts (Diebold and Mariano, 1995). To perform it, we define the forecast errors with nonlinear Markov switching specification (NL) as:

$$\varepsilon^{NL}_{T+h|T} = p_{T+h} - p^{NL}_{T+h|T},$$

where p_{T+h} and $p_{T+h|T}$ are the actual and predicted values of asset prices respectively.

For the linear state-space specification (L):

$$\varepsilon^{L}_{T+h|T} = p_{T+h} - p^{L}_{T+h|T},$$

while for the random walk specification (RW):

$$\varepsilon^{RW}_{T+h|T} = p_{T+h} - p^{RW}_{T+h|T},$$

The loss associated with nonlinear, linear and random walk forecast is assumed to be a function k of the forecast errors: $k\left(\varepsilon^{NL}_{T+h|T}\right)$, $k\left(\varepsilon^{L}_{T+h|T}\right)$ and $k\left(\varepsilon^{RW}_{T+h|T}\right)$ respectively. We denote these functions with the squared-error loss and the absolute value. In this way, for the Markov switching dynamic regression model, we obtain:

$$k\left(\varepsilon^{NL}_{T+h|T}\right) = \left(\varepsilon^{NL}_{T+h|T}\right)^2 \quad and \quad k\left(\varepsilon^{NL}_{T+h|T}\right) = \left|\varepsilon^{NL}_{T+h|T}\right|.$$

For the linear state-space model:

$$k\left(\varepsilon^{L}_{T+h|T}\right) = \left(\varepsilon^{L}_{T+h|T}\right)^2 \quad and \quad k\left(\varepsilon^{L}_{T+h|T}\right) = \left|\varepsilon^{L}_{T+h|T}\right|.$$

For the random walk model:

$$k\left(\varepsilon^{RW}_{T+h|T}\right) = \left(\varepsilon^{RW}_{T+h|T}\right)^2 \quad and \quad k\left(\varepsilon^{RW}_{T+h|T}\right) = \left|\varepsilon^{RW}_{T+h|T}\right|.$$

Considering, for example, the NL and the L models, the loss differential between the two forecasts is:

$$d_{T+h} = k\left(\varepsilon^{L}_{T+h|T}\right) - k\left(\varepsilon^{NL}_{T+h|T}\right).$$

The two forecasts have equal accuracy if the loss differential has zero expectation for all $T+h$. The null hypothesis states that the NL and L forecasts have the same predictive power:

$$H_0 : E\left(d_{T+h}\right) = 0 \quad \forall\left(T+h\right).$$

The alternative hypothesis states that the NL and L forecasts have different levels of performance:

$$H_1 : E\left(d_{T+h}\right) \neq 0.$$

Under H_0, the Diebold-Mariano test statistics is:

$$\frac{\bar{d} - u}{\sqrt{\sigma^2/k_h}} \to N\left(0,1\right).$$

We repeat the test by comparing the three different specifications.

Let us start by comparing the linear state-space model and the random walk hypothesis. Table 3 shows the results obtained. The columns represent the ratio of root mean squared error (RMSE) and the ratio of the mean absolute forecast error (MAE) of the linear model to that of the random walk model. A value greater than 1 indicates the superior performance of the random walk model. The parameter h denotes the forecast horizon in months, and the Diebold-Mariano t-statistics are shown in parentheses.

Let us first examine the RMSE results. From Table 3, we can observe that the behavioral linear model has an inferior performance compared to the random walk forecasts. Overall, the exogenous linear model outperforms the endogenous behavioral linear specification. More in the details, from 1 monthly horizon up to 12 monthly horizon, the RMSE ratio is above the unity and these results are statistically significant at the one percent level according to the Diebold-Mariano test. Passing to the MAE analysis, the ratio is consistently greater than one at the one percent statistical level, indicating significant differences in the predictive accuracy.

To summarize, the random walk model outperforms the linear state-space model in the short-term ($h = 1, 2, 3, 4$ months), medium-term ($h = 5, 6, 7, 8$ months), and long-term ($h = 9, 10, 11, 12$ months) forecasts out-of-sample analysis. Nevertheless, we observe an interesting pattern from Table 3. As the forecasting horizon extends from short-term ($h = 1$ month) to longer-term ($h = 12$ months), the ratio exhibits a consistent downward trend, gradually approaching a value of 1. This trend is indicative of a convergence phenomenon, where the predictive performance of the behavioral linear model increases compared to the random walk. In fact, the superiority of the random walk model seems to get lost in the long run. This is in line with a vision of a linear convergence in the long run to an equilibrium in the system.

Table 3: Out-of-sample forecast results (RW VS LSSM)

h	1 m. h.	2 m. h.	3 m. h.	4 m. h.	5 m. h.	6 m. h.
$RMSE$	4.1931***	4.4996***	3.0120***	1.9908***	1.7901***	1.9683***
	(7.4048)	(5.3465)	(4.5358)	(3.8699)	(4.2157)	(6.6562)
MAE	5.1031***	4.9263***	3.4313***	2.2194***	1.9007***	2.1722***
	(9.9620)	(11.3957)	(8.2932)	(5.9831)	(5.7086)	(7.9130)
h	7 m. h.	8 m. h.	9 m. h.	10 m. h.	11 m. h.	12 m. h.
$RMSE$	2.1504***	2.0635***	1.8062***	1.6582***	1.5737***	1.5606***
	(7.3789)	(5.4001)	(4.5754)	(4.1757)	(4.1679)	(4.5467)
MAE	2.4334***	2.2204***	1.8531***	1.7136***	1.6519***	1.6195***
	(9.0920)	(9.2441)	(6.9737)	(6.1258)	(5.3799)	(6.2541)

Notes: *, **, *** denote statistical significance at the 10%, 5%, and 1% levels respectively.

As a robustness analysis, we repeat the procedure by extending the time horizon from $h = 1$ to $h = 24$ months. Focusing on RMSE (MAE), as evident from Table 4, from the thirteenth (twelfth) to the sixteenth forecast month, the two models are statistically indifferent according to the Diebold Mariano test. Starting from $h = 17$, the ratio is not only below one, but the behavioral linear model also begins to perform better from a statistical point of view.

Table 4: Out-of-sample forecast results (RW VS LSSM)

h	1 m. h.	2 m. h.	3 m. h.	4 m. h.	5 m. h.	6 m. h.
$RMSE$	4.2569***	4.1486***	2.6630***	1.7257***	1.4598***	1.2954***
	(3.6733)	(4.1322)	(4.3764)	(4.5965)	(4.6769)	(4.6701)
MAE	4.4900***	3.8386***	2.5326***	1.7326***	1.4800***	1.4719***
	(5.2658)	(5.1387)	(4.8323)	(4.2422)	(3.6179)	(3.5548)
h	7 m. h.	8 m. h.	9 m. h.	10 m. h.	11 m. h.	12 m. h.
$RMSE$	1.5054***	1.4364***	1.3202***	1.2039***	1.1408***	1.0726*
	(4.2733)	(3.6759)	(3.1017)	(2.7191)	(2.6275)	(1.8340)
MAE	1.4506***	1.3550***	1.2497***	1.1730***	1.1352**	1.0546
	(3.3021)	(3.0224)	(2.5653)	(2.1677)	(2.0089)	(1.0089)
h	13 m. h.	14 m. h.	15 m. h.	16 m. h.	17 m. h.	18 m. h.
$RMSE$	1.0355	1.0180	0.9836	0.9627	0.9169**	0.8927***
	(1.0836)	(0.4310)	(0.4710)	(0.9492)	(2.2076)	(3.2130)
MAE	1.0262	1.0085	0.9834	0.9505	0.9049**	0.8760***
	(0.5570)	(0.1738)	(0.3512)	(1.0365)	(2.1269)	(2.9125)
h	19 m. h.	20 m. h.	21 m. h.	22 m. h.	23 m. h.	24 m. h.
$RMSE$	0.8686***	0.8498***	0.8369***	0.8104***	0.7867***	0.7607***
	(3.9008)	(4.2456)	(4.6387)	(4.7048)	(5.8991)	(6.1051)
MAE	0.8514***	0.8299***	0.8098***	0.7844***	0.7593***	0.7323***
	(3.6706)	(4.3717)	(4.8786)	(5.1302)	(6.1305)	(6.6199)

Notes: *, **, *** denote statistical significance at the 10%, 5%, and 1% levels respectively.

Table 5 presents the outcomes of the out-of-sample forecast comparison between the nonlinear Markov switching dynamic regression model and the linear state-space model. As before, we examine the predictive performance of these two models across different forecast horizons, from $h = 1$ to $h = 12$ months, and considering the ratio of RMSE and MAE of the nonlinear Markov model to the linear state-space model. The results lead to similar conclusions to those reached before. In fact, a clear and noteworthy pattern emerges: the Markov switching model consistently outperforms the behavioral linear model across all forecast horizons, highlighting its ability to capture underlying patterns with greater accuracy. Both for RMSE and MAE metrics, this trend extends across short ($h = 1, 2, 3, 4$ months), medium ($h = 5, 6, 7, 8$ months) and long forecast horizons ($h = 9, 10, 11, 12$ months), implying that the predictive power of the nonlinear model remains robust across different prediction periods at one percent statistical value for the Diebold Mariano test. The obtained results are also confirmed in Table 6, with time horizons ranging from $h = 1$ to $h = 24$.

Overall, the obtained results confirm the importance of considering switching processes in the beliefs of the agents so as to generate nonlinear local endogenous fluctuations phenomena.

Table 5: Out-of-sample forecast results (NLMS VS LSSM)

h	1 m. h.	2 m. h.	3 m. h.	4 m. h.	5 m. h.	6 m. h.
$RMSE$	0.2852***	0.1614***	0.1978***	0.2786***	0.3457***	0.3044***
	(6.7534)	(5.2428)	(4.6059)	(4.1660)	(4.2119)	(6.0283)
MAE	0.2053***	0.1229***	0.1550***	0.2366***	0.2915***	0.2571***
	(8.1815)	(10.1691)	(8.7303)	(7.0922)	(6.4214)	(7.3322)

h	7 m. h.	8 m. h.	9 m. h.	10 m. h.	11 m. h.	12 m. h.
$RMSE$	0.2710***	0.2657***	0.2719***	0.2791***	0.2761***	0.2930***
	(6.6332)	(5.5056)	(4.6879)	(4.3123)	(4.4600)	(3.7747)
MAE	0.2216***	0.2283***	0.2413***	0.2415***	0.2426***	0.2467***
	(8.0314)	(7.4083)	(6.4570)	(5.9369)	(6.1923)	(6.5682)

Notes: *, **, *** denote statistical significance at the 10%, 5%, and 1% levels respectively.

Table 6: Out-of-sample forecast results (NLMS VS LSSM)

h	1 m. h.	2 m. h.	3 m. h.	4 m. h.	5 m. h.	6 m. h.
$RMSE$	0.4668***	0.3452***	0.2921***	0.3532***	0.3448***	0.3178***
	(2.6603)	(4.4794)	(4.1565)	(4.1281)	(4.3798)	(4.8859)
MAE	0.4053***	0.2667***	0.2688***	0.3261***	0.3211***	0.2689***
	(2.9011)	(3.9745)	(4.6464)	(4.6039)	(3.9872)	(3.4979)
h	7 m. h.	8 m. h.	9 m. h.	10 m. h.	11 m. h.	12 m. h.
$RMSE$	0.2666***	0.2577***	0.2516***	0.31269***	0.2779***	0.3164***
	(5.4003)	(5.4667)	(5.6894)	(5.7019)	(6.3467)	(5.5481)
MAE	0.2322***	0.2253***	0.2149***	0.2629***	0.2294***	0.2426***
	(6.3749)	(6.5522)	(7.2560)	(7.1706)	(9.1122)	(6.9906)
h	13 m. h.	14 m. h.	15 m. h.	16 m. h.	17 m. h.	18 m. h.
$RMSE$	0.3583***	0.3458***	0.3542***	0.3774***	0.4192***	0.4303***
	(5.1691)	(5.0523)	(5.3397)	(5.8462)	(5.3308)	(5.3253)
MAE	0.2705***	0.2734***	0.2753***	0.2939***	0.3366***	0.3557***
	(6.3387)	(6.7672)	(6.8316)	(6.9486)	(6.0073)	(5.7261)
h	19 m. h.	20 m. h.	21 m. h.	22 m. h.	23 m. h.	24 m. h.
$RMSE$	0.4477***	0.4327***	0.4212***	0.4413***	0.4437***	0.4592***
	(5.2708)	(5.3679)	(5.13727)	(4.8727)	(5.6722)	(5.4380)
MAE	0.3670***	0.3505***	0.3611***	0.3658***	0.3550***	0.3670***
	(5.5085)	(5.8954)	(5.8530)	(5.5228)	(6.0604)	(5.5807)

Notes: *, **, *** denote statistical significance at the 10%, 5%, and 1% levels respectively.

Finally, we compare the out-of-sample forecasting power of the nonlinear Markov switching dynamic regression model with that of the random walk model. The results in table 7 show that the random walk outperforms the behavioral nonlinear model only for the first short period. Furthermore, for $h = 1$, the RMSE ratio is equal to 1.2787 but not statistically significant for the Diebold Mariano test. Moreover, from the second period onward, the behavioral nonlinear model outperforms significantly better than the random walk model. For $h = 2$, the RMSE ratio equals 0.7277 and is significant at ten percent level. For $h = 3$, the ratio is 0.5852 and statistically significant at five percent level. From $h = 4, ...12$, the ratio tends to decrease, highlighting an increasingly better performance of the nonlinear behavioral model compared to the linear random walk model at one percent level. With respect to MAE, we obtain similar results; for the forecasting horizon $h = 1$ month, the ratio is greater than one, but not statistically significant. The opposite for $h = 2, 3, 4, ..., 12$, where the ratio is smaller than one and statistically significant at one percent level.

Lastly, as done previously, we examine whether the results are valid by extending the forecast time horizon for $h = 1, 2, 3, ..., 24$. From Table 8, the RMSE and MAE ratios are greater than one only for h equal to 1 and 2. Additionally, the results for $h = 2$ are not statistically significant. Starting from the third month, the behavioral nonlinear model begins to outperform significantly, with statistical significance at the 1% level from the fourth month.

To summarize the key finding of our analysis, the proposed NLMS model can beat the behavioral linear model and the exogenous linear model at very different forecasting horizons, both in the short (from $h = 2$), medium and long run.

Table 7: Out-of-sample forecast results (NLMS VS RW)

h	1 m. h.	2 m. h.	3 m. h.	4 m. h.	5 m. h.	6 m. h.
$RMSE$	1.2787	0.7277*	0.5852**	0.5618***	0.6077***	0.5927***
	(1.5954)	(1.8340)	(2.5211)	(3.1686)	(3.1585)	(2.9202)
MAE	1.1521	0.6004***	0.5170***	0.5341***	0.5257***	0.5365***
	(1.0808)	(2.7513)	(2.9773)	(3.4222)	(3.6916)	(3.2631)
h	7 m. h.	8 m. h.	9 m. h.	10 m. h.	11 m. h.	12 m. h.
$RMSE$	0.5808***	0.5545***	0.5110***	0.4518***	0.4469***	0.4767***
	(4.1112)	(3.865)	(3.7063)	(3.6040)	(3.4302)	(3.2952)
MAE	0.5522***	0.5144***	0.4560***	0.4046***	0.4198***	0.4113***
	(2.7485)	(3.3704)	(3.8595)	(4.0082)	(3.8516)	(4.2727)

Notes: *, **, *** denote statistical significance at the 10%, 5%, and 1% levels respectively.

Table 8: Out-of-sample forecast results (NLMS VS RW)

h	1 m. h.	2 m. h.	3 m. h.	4 m. h.	5 m. h.	6 m. h.
$RMSE$	1.9995*	1.5697	0.6892*	0.5794**	0.5168***	0.4778***
	(1.7720)	(0.9527)	(1.8834)	(2.4876)	(2.9382)	(3.7295)
MAE	1.8188**	1.0995	05687**	05143***	0.4675***	0.4035***
	(2.2969)	(0.3131)	(2.4922)	(2.9330)	(3.8318)	(4.8503)
h	7 m. h.	8 m. h.	9 m. h.	10 m. h.	11 m. h.	12 m. h.
$RMSE$	0.4334***	0.3812***	0.3494***	0.3501***	0.3036***	0.3327***
	(3.8083)	(5.4748)	(6.7957)	(6.4451)	(6.8737)	(5.6978)
MAE	0.3889***	0.3112***	0.2844***	0.2756***	0.2417***	0.24876***
	(5.4177)	(8.0171)	(9.8679)	(7.8808)	(8.6207)	(7.5693)
h	13 m. h.	14 m. h.	15 m. h.	16 m. h.	17 m. h.	18 m. h.
$RMSE$	0.3751***	0.3532***	0.3504***	0.3640***	0.3867***	0.3943***
	(5.5958)	(5.8554)	(5.8099)	(6.6745)	(6.5600)	(6.5557)
MAE	0.2987***	0.2745***	0.2698***	0.2787***	0.3070***	0.3283***
	(6.4812)	(6.5942)	(5.5413)	(7.32315)	(6.5002)	(6.5092)
h	19 m. h.	20 m. h.	21 m. h.	22 m. h.	23 m. h.	24 m. h.
$RMSE$	0.3883***	0.3635***	0.3497***	0.3421***	0.3388***	0.3424***
	(6.7698)	(7.1720)	(8.5431)	(8.5305)	(9.7510)	(11.8097)
MAE	0.3217***	0.2999***	0.2840***	0.2789***	0.2623***	0.2671***
	(7.2900)	(8.0374)	(9.5659)	(9.4777)	(10.9280)	(11.7199)

Notes: *, **, *** denote statistical significance at the 10%, 5%, and 1% levels respectively.

5 Conclusions

This paper has analyzed the effect of unobserved speculative strategies of the agents on the determination of the asset price dynamics. To that end, (i) we have transformed the heterogeneous agent setting into a linear state-space form and, based on this, the mathematical condition for complex eigenvalues has been retrieved, (ii) we have moved to a nonlinear Markov setting to possibly highlight local instability phenomena and finally, (iii) we have compared the linear and the nonlinear out-of-sample forecasting power with the benchmark random walk model.

Among our main findings, we have emphasized the presence of speculative behavior which generates endogenous fluctuation in the price formation process. Moreover, the out-of-sample forecasts of the nonlinear heterogeneous agent-based model significantly outperform the forecasts of both the linear heterogeneous agent model and the random walk benchmark model at the short, medium, and long horizons. These findings contribute to the growing body of literature that asserts that financial instability can not be viewed as a simple exogenous phenomenon but as an endogenous process generated by time varying speculative strategies adopted by heterogeneous agents.

Some extensions of the methodology could have interesting results. A direction of future developments ought to consider more than two single behavioral strategies. This would represent a natural step toward a pure agent-based model in which heterogeneity is pervasive in the system. In this sense, the introduction of noise traders or contrarians could be very useful in this respect. Moreover, the analysis was restrained to the equity asset, but its implications could be extended to other financial markets, like the exchange rate. Finally, relying on the same structure, it would be worth moving from a univariate to a multivariate model in which the real side of the economy interacts with the stock market. In this way, we could detect possible correlations between the markets and see how speculative phenomena in financial markets are transmitted to the real sector and vice versa. This is particularly important, especially in the light of an increasing economic-financial integration observed in the last decades.

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