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DISEI
DIPARTIMENTO DI SCIENZE PER L'ECONOMIA E L'IMPRESA

Working Papers - Economics

Monopolistic competition for the market with heterogeneous firms and Schumpeterian growth

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Working Paper N. 09/2019

DISEI, Università degli Studi di Firenze
Via delle Pandette 9, 50127 Firenze (Italia) www.disei.unifi.it

Monopolistic competition *for* the market with heterogeneous firms and Schumpeterian growth

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February 2019

Abstract

I study monopolistic competition in patent races where firms are heterogeneous in R&D costs. Only the most efficient firms invest, and they invest more when the value of innovation is higher, while the endogenous set of active firms depends on the profitability of innovation. In particular, selection effect (increasing R&D productivity) emerge after a reduction of the entry cost or after an increase (a reduction) of the value of innovation if the elasticity of the probability of innovation is increasing (decreasing) in investment. In Schumpeterian models selection effects foster endogenous growth.

Key words: Patent races, heterogeneous firms, monopolistic competition, Schumpeterian growth.

JEL Code: L1, O3, O4.

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In most sectors only some firms invest in innovation, R&D intensity varies a lot between firms, and most of the important patents are typically concentrated in the hands of few large firms, though also small ones occasionally innovate (for a survey on innovative activities see Scotchmer, 2004). These basic facts are at odds with theories of innovation based on Nash competition between identical firms, as in symmetric patent races in partial equilibrium (since Loury, 1979; Lee and Wilde, 1980; Dasgupta and Stiglitz, 1980) and in standard Schumpeterian growth models (since Aghion and Howitt, 1992). Strategic and technological advantages for incumbent patent holders can explain differences in R&D activity between incumbents and outsiders in patent races, but not the large differences within each category. In this work I study the implications of cost heterogeneity among firms under monopolistic competition to match the mentioned facts and analyze the impact of shocks on the R&D investment of different firms and on the set of firms engaged in R&D activity.

In particular, I study patent races with invetment flows determining the arrival rate of innovations (Lee and Wilde, 1980; Reinganum, 1985) introducing monopolistic competition behavior *à la* Dixit and Stiglitz (1977), rather than strategic behavior, and considering free entry of firms that are heterogeneous in R&D costs in the spirit of Melitz (2003).[2] Participation to the patent race requires a fixed entry cost and, once firms draw a productivity parameter for their R&D technology, they decide whether to pay a fixed cost and start the R&D

[2] I apply recent advances in the theory of monopolistic competition in the market (Bertoletti and Etro, 2016, 2017, 2018). Similar applications of monopolistic competition are suggested in aggregative games as those analyzed by Etro (2006) and Anderson, Erkal and Piccinin (2019). For a recent application to endogenous growth models see Etro (2019), whose focus is however on monopolistic competition in the market for the production of intermediate inputs.

activity. This implies that only some firms do invest in R&D and the investment is chosen differently by each one, with larger investment, and higher probability of innovation, for more productive firms. Moreover, changes in profitability in the market, in IPR protection or in the R&D subsidies induce selection effects on which firms do invest. For instance, an increase of the fixed costs of R&D or a reduction of the entry cost induce always a selection effect. Instead, the impact of changes in the value of innovation (for instance due to changes in R&D subsidies or opening up to new markets) is ambiguous: my main result is that an increase in the value of innovation increases the investment of each active firm, but reduces (increases) the cost cut-off to be engaged in R&D if the elasticity of the probability of innovation is always increasing (decreasing). This implies that the productivity of an R&D sector can either increase or decrease when the innovations become more profitable depending on technological conditions.

I extend the analysis of the patent race a) with an incumbent patent holder, which is inconsequential on the results under monopolistic competition (the incumbent invests as the outsiders), b) with endogenous size of innovations, showing that more efficient firms invest also in more valuable innovations, and c) with fixed costs of R&D determining the probability of innovation (Loury, 1979), in which case the value of innovation becomes neutral not only on the set of active firms but even on the investment of the active firms.

Finally, I apply the analysis to a Schumpeterian growth model with heterogeneous firms engaged in sequential patent races showing how selection effects foster growth. Recent endogenous growth models with heterogeneous firms include those by Sampson (2016), Perla *et al.* (2018) and Haruyama and Zhao

(2018),[3] and analyze selection effects induced by trade liberalization. However, they focus on heterogeneous firms in the production sector *à la* Melitz (2003) and not in the market for innovations as here. Classic Schumpeterian models *à la* Aghion and Howitt (1992) are based on arrival rates of innovation that are linear in the aggregate flow of R&D spending (no-arbitrage conditions determine aggregate investment flows), therefore they are not appropriate to analyze monopolistic competition for the market with heterogeneity of firms and fixed costs of R&D. Building on patent races with decreasing marginal productivity of R&D spending and fixed costs (as in Etro, 2004), I can analyze the endogenous market structures of sequential monopolistic competition for the market and the role of heterogeneity, focusing on an example with a power function for the probability of innovation and a Pareto distribution for the productivity parameter. The novel implication is that growth is fostered by selection effects on the set of firms engaged in R&D, in particular when there is a reduction in the entry cost. In an open economy framework, this may be due to a reduction of the set up costs needed to be able to patent, commercialize and export new goods abroad in case of innovation, therefore as long as trade liberalization reduces these entry costs, it contributes to increase R&D productivity and growth. Instead, a reduction of the fixed cost of R&D has the opposite effect of allowing entry of less efficient firms.

I present the baseline model in Section 1. In Section 2 I discuss extensions with an incumbent patent holder, endogenous size of innovations, contractual costs of R&D and I comment on the welfare analysis. I finally apply the model to endogenous growth in a general equilibrium model with sequential patent

[3] See also Baldwin and Robert-Nicoud (2008), Gustafsson and Segerstrom (2010) for earlier contributions in this direction.

races in Section 3 and conclude in Section 4.

1 A patent race with heterogeneous R&D costs

Consider a stochastic patent race in the continuum, where firms invest over time to obtain an innovation of value $V > 0$. Participation to the patent race requires an entry cost $F_e > 0$, which can be interpreted as a sunk cost of entry in the market without an innovation, a cost of preparing IPRs and commercialization for the innovative products or an initial investment in basic R&D without reference to the particular innovation. Entrants draw a cost parameter c from a known distribution $G(c)$ on the support $[0, \bar{c}]$ for some finite $\bar{c} > 0$, and decide whether to pay an additional fixed cost $F > 0$ to start R&D, which can be interpreted as a cost to build a lab and hire researcher for the specific innovation. Each active firm ι with cost c invests a flow $z(\iota)$ and obtains the innovation according to a Poisson process with arrival rate $h(z(\iota))$, with $h(z) > 0$, $h'(z) > 0$ and $h''(z) < 0$ for any $z > 0$. As an example I will consider the power function $h(z) = (\phi z)^{\varepsilon}$ where $\varepsilon < 1$ represents the (constant) elasticity of the probability of innovation (as well as the elasticity of the expected revenues) with respect to investment, and $\phi > 0$ is a scale parameter (possibly depending on the characteristics of the sector or of the innovation, as in the model with multiple patent races of Section 3).

Given a constant interest rate r, the expected profits of a firm ι are:

$$\pi(\iota) = \frac{h(z(\iota))V - cz(\iota)}{r + p} - F \tag{1}$$

where the aggregate arrival rate of innovations by any firm in the market is:

$$p = \int_{\Omega} h(z(\omega))d\omega$$

with Ω defined as the set of active firms where $z(\omega)$ is the investment of firm $\omega \in \Omega$.

Each firm decides the investment flow taking as given the aggregate arrival rate of innovations in the spirit of Dixit and Stiglitz (1977), on the basis of the first order condition:

$$h'(z)V = c \tag{2}$$

This defines the equilibrium investment $z(c)$ as a decreasing function of c and an increasing function of V since:

$$\frac{d \ln z}{d \ln V} = -\frac{d \ln z}{d \ln c} = \frac{1}{\sigma(z)} > 0 \tag{3}$$

where $\sigma(z) \equiv -h''(z)z/h'(z) > 0$ is the elasticity of the marginal probability of innovation. More efficient firms have a higher probability of innovating and becoming incumbents in the industry. The monopolistic behavior implies that the mass of entrants N and the interest rate r, as well as the fixed costs, do no affect the investment flow of each firm. Notice that this is not the case when strategic interactions are taken in consideration (Lee and Wilde, 1980; Reinganum, 1985), in which case each firm invests less (since it internalizes the impact of its investment on the aggregate probability of innovation).[4] In the case of the mentioned power function the equilibrium investment is $z(c) = \left(\frac{\varepsilon\phi^{\varepsilon}V}{c}\right)^{\frac{1}{1-\varepsilon}}$ with $\sigma(z) = 1 - \varepsilon$.

Since all firms with cost draws below a cut-off $\widehat{c}$ are engaged in R&D, the

[4] Under free entry with homogeneous firms (Etro, 2004) the Nash equilibrium satisfies $h'(z)(V - F) = c$ confirming a lower investment. Nevertheless, Stackelberg leaders facing endogenous entry would adopt the same investment (2) as in the monopolistic competiton equilibrium.

zero profit condition:

$$\frac{h(z(\widehat{c}))V-\widehat{c}z(\widehat{c})}{r+N\int_0^{\widehat{c}}h\left(z\left(t\right)\right)dG\left(t\right)}=F$$

defines this cut-off in function of the mass of entrants as well as of the other exogenous variables. The mass of active firms can be derived as $n\equiv NG(\widehat{c})$: inefficient firms do not invest in R&D and most innovations derive from the most efficient firms.

This implies the *ex ante* expected profits:

$$\mathbb{E}\left[\pi\right]=\int_0^{\widehat{c}}\frac{h(z(t))V-tz(t)}{r+N\int_0^{\widehat{c}}h\left(z\left(t\right)\right)dG\left(t\right)}dG\left(t\right)-FG(\widehat{c})$$

The free entry condition equates this to F_e. Such a condition can be rewritten through the zero profit condition as follows:

$$\frac{\int_0^{\widehat{c}}\left[h(z(t))V-tz(t)\right]dG\left(t\right)}{h(z(\widehat{c}))V-\widehat{c}z(\widehat{c})}=G(\widehat{c})+\frac{F_e}{F}\tag{4}$$

which identifies alone the equilibrium cut-off equating the ratio between average profit flow $\mathbb{E}\left[h(z(c))V-cz(c)\right]$ and marginal profit flow $h(z(\widehat{c}))V-\widehat{c}z(\widehat{c})$ to the sum of the fraction of active firms $G(\widehat{c})$ and the ratio of entry and fixed costs F_e/F.

The cut-off is independent from the interest rate r, but depends on the value of innovations and the fixed costs.[5] An increase of the entry cost F_e increases the right hand side of (4) inducing selection (a reduction of $\widehat{c}$), while an increase in the fixed cost of R&D F has the opposite impact. I summarize these results as follows:

Proposition 1. *Under monopolistic competition for the market with heterogeneous firms, a reduction of entry costs (an increase of fixed R&D costs) is*

[5] Since $\frac{\mathbb{E}[h(z(c))V-cz(c)]}{h(z(\widehat{c}))V-\widehat{c}z(\widehat{c})}-G(\widehat{c})$ is increasing in $\widehat{c}$, the sign of the impact of fixed costs on $\widehat{c}$ is the opposite of the sign of the impact on the the right hand side of (4).

neutral on the investment of each active firm in R&D and reduces (increases) the cost cut-off to be engaged in R&D, while a change of the interest rate is neutral on both investment of each firm and cut-off.

The effect of an increase in the value of innovation V is positive on the investment of each firm but ambiguous on the set of active firms. To characterize the direction of its impact, let me define the elasticity of the probability of innovation (which is also the elasticity of the expected revenues) as $\varepsilon(z) \equiv h'(z)z/h(z) > 0$, with derivative:

$$\varepsilon'(z) \propto 1 - \varepsilon(z) - \sigma(z)$$

that can be either positive or negative, as well as constant, as in case of a power function.

Now, the left hand side of (4) is always increasing in V if its numerator increases more than the denominator when the value of innovation increases, in which case the increase in V reduces $\widehat{c}$. This is always the case if:

$$\left[\int_0^{\widehat{c}} h(z(t))dG(t)\right][h(z(\widehat{c}))V - \widehat{c}z(\widehat{c})] > h(z(\widehat{c}))\left[\int_0^{\widehat{c}} [h(z(t))V - tz(t)]\, dG(t)\right]$$

or:

$$\frac{h(z(\widehat{c}))}{\widehat{c}z(\widehat{c})} > \frac{\int_0^{\widehat{c}} h(z(t))dG(t)}{\int_0^{\widehat{c}} tz(t)dG(t)}$$

This condition is always satisfied if the function $\psi(c) \equiv h(z(c))/cz(c)$ is increasing in the marginal cost. But using (3) we can compute:

$$\psi'(c) \propto 1 - \varepsilon(z(c)) - \sigma(z(c)) = \varepsilon'(z(c))$$

which delivers the following result:

PROPOSITION 2. *Under monopolistic competition for the market with heterogeneous firms, an increase in the value of innovation increases the investment of* *each active firm in R&D and reduces (increases) the cost cut-off to be engaged in R&D if the elasticity of the probability of innovation is always increasing (decreasing) in investment.*

Intuitively, when the probability of innovation becomes more elastic with higher investment, the expected return on investment, which depends on the ratio between probability of innovation and investment $\psi(c)$, is lower for firms investing more, and the increase in investment associated with more valuable innovations makes it harder to break even, which induces the exit of the high cost firms (a selection effect). Instead, when the probability of innovation becomes less elastic with the size of the investment, more efficient firms have a higher expected return on investment and more valuable innovations relax the selection constraint.

Only in the special case of a constant elasticity, the value of innovations are neutral. Indeed, with the mentioned power function the cut-off is defined by the condition $\int_0^{\widehat{c}} \left(\frac{\widehat{c}}{t}\right)^{\frac{\varepsilon}{1-\varepsilon}} dG(t) = G(\widehat{c}) + \frac{F_e}{F}$, which is independent from V (as well as from the scale parameter ϕ of the probability of innovation) for any cost distribution. It can be useful to consider the case of a Pareto distribution for the cost parameter, such as:

$$G(c) = \left(\frac{c}{\bar{c}}\right)^{\kappa}$$

with shape parameter κ assumed to satisfy $\kappa > \frac{\varepsilon}{1-\varepsilon}$. In such a case I can integrate and solve explicitly for the cut-off as:

$$\widehat{c} = \left[\frac{F_e}{F}\left(\frac{\kappa(1-\varepsilon)}{\varepsilon} - 1\right)\right]^{\frac{1}{\kappa}} \bar{c} \qquad (5)$$

which is decreasing (i.e. making R&D more productive) in the fixed cost of R&D relative to the fixed cost of entry, in the elasticity of the probability of innovation ε, and when the cost distribution is more uniform (lower κ for low values).

The equilibrium mass of entrants derives from the zero profit condition as follows:

$$N = \frac{h(z(\widehat{c}))V - \widehat{c}z(\widehat{c}) - rF}{F\int_0^{\widehat{c}} h\left(z\left(t\right)\right) dG\left(t\right)} \tag{6}$$

which closes the model. Clearly an increase in the interest rate exerts its effect on R&D only through the reduction of the mass of entrants. In case of a power function and a Pareto distribution we can also compute the mass of firms as:

$$N = \frac{\kappa - \frac{\varepsilon}{1-\varepsilon}}{\kappa}\left[\frac{\left(1-\varepsilon\right)V}{F} - r\left(\frac{\widehat{c}}{\varepsilon\phi V}\right)^{\frac{\varepsilon}{1-\varepsilon}}\right]\left(\frac{\bar{c}}{\widehat{c}}\right)^{\kappa}$$

where $\widehat{c}$ is given by (5). When the value of innovation increases (for instance due to opening up to trade with countries protecting IPRs) entry increases. With a zero interest rate the impact would be linear, but a positive interest rate reduces investment in business creation and makes it more sensitive to changes in the value of innovation. Similarly, selection effects have an impact on entry: for instance, a reduction in entry costs promotes entry while restricting the conditions for being active, with a positive net impact on the mass of active firms.

It is time to pause to consider the implications of the main result. Consider a strengthening of IPRs that augments the value of an innovation, or opening up to trade with countries where the same innovations are protected, or the introduction of a subsidy to variable R&D spending. The impact of all these changes is equivalent to the impact of an increase in V in our model,[6] and

[6] Consider an R&D subsidy on variable investment at rate s and a subsidy on fixed spending τ such that expected profits become:

$$\pi(\iota) = \frac{h(z(\iota))V - (1-s)cz(\iota)}{r+p} - (F-\tau)$$

it induces a selection effect if $\varepsilon'(z) > 0$: in spite of the increase in potential profitability for any investor, less efficient firms will have to stop investing, leaving the most efficient ones in the patent race. Of course, this increases R&D productivity. Whether this case is more realistic is an empirical question that deserves further investigation (going beyond the traditional estimates of the elasticity ε assumed constant).[7]

For aggregate purposes, such as analyzing the rate of technological progress, it is also important to look at the aggregate probability of innovation. This can be expressed as:

$$p = \frac{h(z(\widehat{c}))V - \widehat{c}z(\widehat{c})}{F} - r \tag{7}$$

which is decreasing in the cut-off, in the interest rate and in the fixed cost. The impact of the value of innovation $\frac{\partial p}{\partial V} = \frac{h(z(\widehat{c}))}{F} - \frac{z(\widehat{c})}{F}\frac{\partial \widehat{c}}{\partial V}$ is always positive in the presence of selection effects. In case of a power function I have:

$$p = (1-\varepsilon)\left(\frac{\varepsilon\phi}{\widehat{c}}\right)^{\frac{\varepsilon}{1-\varepsilon}} \frac{V^{\frac{1}{1-\varepsilon}}}{F} - r \tag{8}$$

where $\widehat{c}$ is given by (5) under a Pareto distribution.

2 Extensions

In this section I will briefly discuss straightforward extensions of the baseline model. First of all, it is natural to ask how the analysis above changes when

This changes the optimality condition into $h'(z)V = (1-s)c$, so that investment and cut-off depend on $V/(1-s)$, while the fixed subsidy affects only the cut-off.

[7] Selection effects in the value of innovation emerge naturally with a translated power function such as $h(z) = (\phi + z)^{\varepsilon}$, where $\phi > 0$ now parametrizes the arrival rate of innovations without variable investment. In such a case $\varepsilon'(z) > 0$ and more valuable innovations induce selection effects.

there is an incumbent patent holder, with a flow of profits $\bar{\pi}$ which stops at the time of the new innovation. In standard patent races with strategic interactions, such a patent holder has lower incentives to invest in R&D due to the Arrow effect (its innovation eliminates an on-going rent) and in case of free entry has no incentives to invest at all. Only in case of a leadership in the patent race, the incumbent can have higher incentives to invest due to a Stackelberg effect, and this is always the case under free entry (its investment crowds out entry).[8] The scenario is quite different in our environment with monopolistic competition.

Consider the expected profits of the incumbent patent holder with a given cost c:

$$\pi^I = \frac{h(z^I)V + \bar{\pi} - cz^I}{r+p} - F \tag{9}$$

where $\bar{\pi} > 0$ is the flow of monopolistic profits from the patent of the incumbent. In case of positive investment, the incumbent follows the same investment rule as before, with $h'(z^I)V = c$. This implies that the cut-off for the incumbent to be active is also the same as for all the outsiders. Accordingly, the equilibrium of the patent race is unchanged. In practice both the Arrow effect and the Stackelberg effect vanish under monopolistic competition with heterogeneous firms. As a consequence, our core analysis applies naturally to sequential innovations, as I will show in the next section.

One can consider an endogenous size of innovations by introducing and additional investment flow $x(\iota)$ for firm ι determining the value of innovation $V(x(\iota))$ with $V'(x) > 0$ and $V''(x) < 0$. In such a case the expected profits change to:

$$\pi(\iota) = \frac{h(z(\iota))V(x(\iota)) - cz(\iota) - x(\iota)}{r+p} - F \tag{10}$$

[8] The Arrow effect goes back to Arrow (1962). A detailed analysis of these results can be found in Etro (2004). For evidence see for instance Czarnitzki *et al.* (2014).

and the additional first order condition:

$$h(z)V'(x) = 1 \tag{11}$$

determines jointly with (2) the two investment flows $z(c)$ and $x(c)$. The second order conditions for a relative maximum require also $\sigma(z)\varphi(x) > \varepsilon(z)\mu(x)$ where $\mu(x) \equiv \frac{V'(x)x}{V(x)} > 0$ and $\varphi(x) \equiv -\frac{V''(x)x}{V'(x)} > 0$ are the elasticities of the absolute and marginal value of innovation. Low-c firms are more efficient in obtaining innovations, and also relatively more efficient in this rather than in obtaining more valuable innovations. Nevertheless, complementarity of the two investment stategies implies that more efficient firms invest also in more valuable innovations. Indeed, total differentiation provides:

$$\frac{d\ln z}{d\ln c} = \frac{-\varphi(x)}{\sigma(z)\varphi(x) - \varepsilon(z)\mu(x)} \quad \text{and} \quad \frac{d\ln x}{d\ln c} = \frac{-\varepsilon(z)}{\sigma(z)\varphi(x) - \varepsilon(z)\mu(x)}$$

which are both negative under our assumptions. Since expected profits remain monotonically decreasing in the cost parameter, the equilibrium cut-off and the mass of entrants are derived as before.[9] Summing up, larger firms are more likely to innovate and, when they innovate, they also obtain innovations of larger value.

Our main result on the impact of the value of innovation depends on the nature of the R&D technology, and in particular on the fact that the arrival rate of innovations depends on the simultaneous flow of variable R&D investment. To appreciate this, consider the case of a context *à la* Tullock or a patent

[9] For instance, with power functions $h(z) = (\phi z)^{\varepsilon}$ and $V(x) = x^{\mu}$ where $\mu \in (0, 1-\varepsilon)$ we obtain:

$$z(c) = \left[\mu\phi^{\varepsilon}\left(\frac{\varepsilon}{c}\right)^{1-\mu}\right]^{\frac{1}{1-\mu-\varepsilon}} \quad \text{and} \quad x(c) = \mu\left(\frac{\varepsilon\phi}{c}\right)^{\frac{\varepsilon}{1-\mu-\varepsilon}}$$

These are both decresing in c, and changes in the scale parameter ϕ remain neutral on the cut-off.

race with contractual costs of R&D (Loury, 1979), in which the arrival rate of innovations depends on the initial fixed investment. The expected profits of an entrant becomes:

$$\pi(\iota) = \frac{h(z(\iota))V}{r+p} - cz(\iota) - F \tag{12}$$

where F is always the exogenous fixed cost and $cz(\iota)$ is the endogenous fixed investment by firm ι chosen after drawing the cost parameter c and deciding to be active.

In such a case, the equilibrium investment of the active firms increases in their productivity, but the cut-off for being active and even the investment of each firm are independent from the value of innovation. Indeed, the investment rule and the cut-off satisfy:[10]

$$\frac{h'(z)[\widehat{c}z(\widehat{c}) + F]}{h(z(\widehat{c}))} = c$$

and

$$\int_0^{\widehat{c}} \left[\frac{h(z(t))}{h(z(\widehat{c}))} \left[1 + \frac{\widehat{c}z(\widehat{c})}{F} \right] - \frac{tz(t)}{F} \right] dG(t) = G(\widehat{c}) + \frac{F_e}{F} \tag{13}$$

independently from V, and the mass of entrants becomes:

$$N = \frac{h(z(\widehat{c}))V - r\left[\widehat{c}z(\widehat{c}) + F\right]}{\left[\widehat{c}z(\widehat{c}) + F\right] \int_0^{\widehat{c}} h\left(z\left(t\right)\right) dG\left(t\right)}$$

Intuitively, an increase in the value of innovation increases the number of competitors, leaving unchanged the absolute and marginal profitability of each firm. Instead, a reduction of the entry cost induces a selection effect as before.

A last consideration on welfare. In our model investment is not optimal for the simple reason that the social value of innovation differs from the private value that drives the equilibrium. Moreover, even assuming equality of private

[10] Notice that the investment of each active firm is still decreasing in c but it now depends on the entry and fixed costs.

and social value of the innovation, the optimal allocation that selects $z(c)$, $\widehat{c}$ and N to maximize total surplus:

$$TS = N\left[\frac{\int_0^{\widehat{c}}\left[h(z(t))V - tz(t)\right]dG\left(t\right)}{r + N\int_0^{\widehat{c}} h\left(z\left(t\right)\right)dG\left(t\right)} - FG(\widehat{c}) - F_e\right]$$

is not the same as the equilibrium one.[11] The reason is that under monopolistic competition each investor does not take in consideration the impact of its entry choice and its decision to start investing on the aggregate probability of innovation (therefore, even under a power function for the arrival rate of innovations, too many firms are active in R&D).

3 Schumpeterian growth

The model can be applied to Schumpeterian theories of endogenous growth (Aghion and Howitt, 1992) where value of sequential innovations, interest rate and growth rate are endogenized in general equilibrium. A recent literature has introduced heterogeneous firms in this class of models with horizontal innovation *à la* Romer (1990), as in Baldwin and Robert-Nicoud (2008), Gustafsson and Segerstrom (2010) and Sampson (2016) or vertical innovation *à la* Aghion and Howitt (1992) as in Haruyama and Zhao (2018), but focusing on the market for goods rather than on the market for innovations.[12]

Here I follow the model of Barro and Sala-i-Martin (2004) extended to patents races with concave functions for the arrival rate of innovations as in Etro (2004), and augmented with monopolistic behavior by heterogeneous firms.

[11] A system of R&D subsidies and entry fees can be used to restore optimality. Optimality in related models of monopolistic competition with heterogeneous firms is discussed in Bertoletti *et al.* (2018) and Bertoletti and Etro (2017).

[12] See Feenstra (2016) for a good survey of this literature.

Consider an infinite horizon model in the continuum, with a representative agent with logarithmic utility and preference rate ρ, so that the growth rate of consumption is given by $g = r - \rho$ in a stationary environment, and a production function of final goods:

$$Y = (AL)^{1-\alpha} \sum_{j=1}^{n} (q^{k_j} X_j)^{\alpha} \tag{14}$$

where X_j is the amount of intermediate good $j = 1, 2, ..., n$ of quality k_j, $\alpha \in (0, 1)$ is the factor share of these inputs, $q > 1/\alpha$ parametrizes the size of each innovation in terms of productivity, A is constant labor productivity and L is the labor input supplied in a fixed quantity. The final good is produced competitively and is the *numeraire* of the economy, employed for consumption, production of intermediate goods and R&D spending.

The intermediate goods of the best quality are produced by monopolistic patent holders, and patent races as those analyzed above take place to create the next quality of each intermediate good. As mentioned above, the incumbent patent holders participate to the race for the next innovation and, without assuming any technological advantage, they follow the same investment rules as the other firms. This is inconsequential on the aggregate behavior of the economy under monopolistic competition.

It is standard to derive the demand of each intermediate good and its monopolistic price as a constant $1/\alpha$, implying production levels X_j and profit flows $\frac{1-\alpha}{\alpha} X_j$ increasing with the quality of the input j, and a present discounted value of innovation:

$$V_j = \frac{1-\alpha}{\alpha} \frac{X_j}{r+p}$$

in a stationary environment. This requires increasing costs of innovation, and as in Etro (2004) I assume a power function for the probability of innovation with scale parameter $\phi = 1/X_j$, and fixed costs of production and entry that are fractions respectively η and f of the expected costs of production $\frac{X_j}{r+p}$. Then I can solve (8) as:

$$\begin{aligned} r+p &= (1-\varepsilon)\left(\frac{\varepsilon}{\widehat{c}}\right)^{\frac{\varepsilon}{1-\varepsilon}} \frac{\left(\frac{1-\alpha}{\alpha(r+p)}\right)^{\frac{1}{1-\varepsilon}}}{\left(\frac{\eta}{r+p}\right)} = \\ &= \frac{1-\alpha}{\alpha}\left(\frac{1-\varepsilon}{\eta}\right)^{1-\varepsilon}\left(\frac{\varepsilon}{\widehat{c}}\right)^{\frac{\varepsilon}{1-\varepsilon}} \end{aligned}$$

As in Barro and Sala-i-Martin (2004), the growth rate of output must be equal to the growth rate of consumption and to the expected increase in productivity $g = p[q^{\frac{\alpha}{1-\alpha}} - 1]$, which allows me to solve it as follows:[13]

$$g = \frac{\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\varepsilon}{\widehat{c}}\right)^{\varepsilon}\left(\frac{1-\varepsilon}{\eta}\right)^{1-\varepsilon} - \rho}{1 + [q^{\frac{\alpha}{1-\alpha}} - 1]^{-1}} \tag{15}$$

where the equilibrium cut-off is:

$$\widehat{c} = \left[\frac{f}{\eta}\left(\frac{\kappa(1-\varepsilon)}{\varepsilon} - 1\right)\right]^{\frac{1}{\kappa}} \bar{c} \tag{16}$$

under the Pareto distribution.

As usual, Schumpeterian growth increases in the market power of the innovators (i.e. the markup $\frac{1-\alpha}{\alpha}$) and in the quality increase (q), and decreases in the preference rate (ρ). A new aspect is that the growth rate increases when the cut-off $\widehat{c}$ decreases: the more selective is the market for innovation, the more productive are the active firms and the higher is growth, and the other way around. For instance, a reduction of the fixed costs of innovation (a reduction of η) has a direct positive effect on growth (the aggregate probability of innovations increases), but this is in part compensated by a selection effect (new

[13] Notice that with homogeneous firms ($\kappa \to \infty$) the growth rate differs from the one in Etro (2004) due to the Nash behavior, and with also a linear R&D technology ($\varepsilon \to 1$) it is in line with the one of Barro and Sala-i-Martin (2004).

inefficient firms become active and the average productivity of the R&D sector decreases).

The main novel result is that a reduction of the entry costs (a reduction of f) prevents the most inefficient firms from investing and increases the productivity of the R&D sector, which in turn raises the growth rate. Since the entry cost affects growth only through the selection effect, this is an effect that is absent in models with homogeneous firms. In an open economy interpretation of the model, a reduction in the entry cost may be due to a reduction of the set up costs needed to be able to patent, commercialize and export new goods abroad in case of innovation, therefore as long as trade liberalization reduces these entry costs, it contributes to increase R&D productivity and growth.

Finally, notice that in this example, market expansion associated with an increase in the value of innovations does not affect directly the selection of firms active in R&D due to the power function for the arrival rate of innovations. Departing from this with a general function for the probability of innovation, also market expansion would affect the productivity of the R&D sector, with further feedbacks on growth. In particular the equilibrium probability of innovation and cut-off satisfy (4) and (7) given the endogenous value of innovation and the functional forms for scale parameters and fixed costs. Then, when the elasticity of the probability of innovation is increasing in investment, an increase in the value of innovation induces selection effects that foster growth.

4 Conclusion

I have analyzed monopolistic competition for the market where firms are heterogeneous in R&D costs. This implies that only some firms do invest and they invest more when the value of innovation is higher, but market expansion or R&D policy induce ambiguous selection effects à la Melitz. In Schumpeterian models, these selection effects affect the growth rate. It would be interesting to apply the analysis to fully fledged open economy models with trade costs to identify channels through which trade liberalization affects the innovative activity and growth.

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