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Market dynamics with a state-owned dominant firm and a competitive fringe

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Abstract

We analyze the market dynamics in a model in which one dominant firm and a large number of small, not fully rational firms coexist. The dominant firm announces a reference price, but the market price can diverge from such reference price: this is due to the dominant firm taking advantage of the bounded rationality of the fringe firms. In the baseline model, we find that the dominant firm has an incentive to announce a very low price and in the steady state the market price is usually higher than the reference price. In a more complex model, where a fraction of small firms employ an evolutionary mechanism to adjust their expectations, we find that the lower the reference price the higher the period-by-period fluctuations of the market prices. We show that both mean profits and their volatility are decreasing in the reference price and that the optimal choice is positively correlated with the degree of risk aversion of the dominant firm. In general, socially preferable outcomes can be achieved when the dominant firm behaves as strongly risk averse. We draw some policy implications from this conclusion.

1 Introduction

After the second world war in many European countries the public sector was directly involved in the management of many public services. For instance, energy, railway, telephone were frequently industries where a State owned monopolist was in charge of the provision of such services under the supervision of some political committee. However, in the last decades of the previous century, under the pressure of stagflation and increasing public debt, many countries began a process of privatization and liberalization in many of these sectors in order to stimulate economic growth.[1] When possible, Governments tried to substitute former public monopolies with a large number of private firms, following the idea that a higher competition in such sectors could achieve better results in the view of public interest. From a theoretical point of view, such policy was in line with the widespread opinion held by many economists who emphasized the inefficiencies associated with public monopolies (lack of incentives to innovation and to cost reduction) and the potential advantages of competition. However, due to technological and institutional factors such industries very often could not be structured in a way similar to perfect competition because the market share of the incumbent firm remained relevant.[2] Moreover, in some cases the former public monopolist was not fully privatized, and for these reasons the dominant firm in the sector continued to be influenced by its special property regime.[3] Therefore, in order to better understand the implications of these reforms, it is important to thoroughly analyze market settings characterized by both imperfect competition and the presence of a (at least partially) state-owned enterprise. In the economic literature various papers deal with “mixed” oligopoly, that is an oligopoly in the presence of a state-owned firm that is, at most, only partially privatized (see e.g., [9, 15, 21, 22]). Conversely, to the best of our knowledge, there is no theoretical literature regarding a market configuration where a unique state-owned firm is in dominant position in its sector and faces a competitive fringe of private firms. There is instead a wide literature studying models where a privately owned dominant firm and a large number of small firms coexist.

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[1] [24] and [3] emphasize how privatization and liberalization boosted firms' investments and productivity growth in network industries as telecommunication, railway and electricity.

[2] For instance, this fact frequently happened in telephone and energy production sectors, see [2], [6], [7], and [18].

[3] See for instance the energy sector in many European countries, as emphasized in [17] focusing on the case of France.

As illustrated in [25], the first analysis of this kind of market dates back to before the second world war, but the most complete presentation can be found in [26]. In such model, small firms are price-takers and all together constitute a competitive fringe, while the dominant firm behaves as a standard price-maker monopolist whose demand corresponds to the residual demand, i.e. at any price, the market demand diminished by the quantity produced by the competitive fringe at that price. As emphasized in [25], this kind of model is based on a static equilibrium analysis and relies on various assumptions which have been relaxed in subsequent papers. For instance, [5, 14] have removed the assumption of an exogenous number of small firms, allowing for market entry and exit depending on achievable profits. In addition, the mechanism by which dominant and small firms coordinate their choices in this static version of the market is left implicit. The general idea is that the dominant firm announces a price, each small firm chooses the optimal quantity given that price and the dominant firm produces a quantity equal to the residual demand at that price so that eventually the market-clearing price coincides with the announced one. Whether the dominant firm will find it convenient to engage in the production of a quantity such as to ensure the coincidence between announced price and actual price, though, is not analyzed. In order to do so it is necessary to embed this market structure in a dynamic framework.

In the literature there have been various attempts to enlarge the analysis of the dominant firm to a dynamic version. [29] and [20] tried to explain the observed phenomenon of the reduction of the dominant firm market share over time. A dynamic analysis of the limit pricing strategy in a framework where the rate of entry is proportional to the distance between the current price and limit price is performed in [16] whose conclusions hinge inextricably on the assumption of fringe firms having adaptive expectations about future prices. This result is disputed by [5] showing that under rational expectations, the conclusions would be quite different. Most of these papers have assumed that at each time period the dominant firm always chooses a quantity such that the actual price is coincident with its price announcement. An exception is [28], where it is assumed that the price announcement of the dominant firm can be a strategic decision in order to influence rival firms' reactions and that the dominant firm is not constrained to produce a quantity consistent with such announcement. However, in this paper there is only one rival, perfectly rational firm, and for this reason the dominant firm cannot deceive its rival (while concealment of private information remains possible).

In the present paper we focus on the strategic choices of the dominant firm in a setup where (1) fringe firms adjust their choices forming expectations about future prices using simple heuristics, and (2) the dominant firm maximizes profits in the current period perfectly anticipating fringe firms production choices. We assume that at the beginning of the game the dominant firm announces a benchmark price. After this preliminary stage there is an infinite repetition of the following stage-game: on the basis of the benchmark price and of the previous price each small firm forms expectations regarding the price of the current period and consequently chooses how much to produce. The dominant firm anticipates the fringe aggregate output production and, after calculating the residual demand, chooses how much to produce in order to maximize its current period profits.

In the baseline version of the model we assume that the firms in the competitive fringe form expectations as a convex combination of the price announced by the dominant firm and the price observed in the previous period. In this context we obtain that the dominant firm has a clear incentive to announce as low a price as possible in order to reduce the quantity supplied by the competitive fringe firms. However, this result is related to the occurrence of a stationary equilibrium in which the competitive fringe firms expect a price that is always different from the market price. Moreover, this equilibrium is dynamically stable only under specific technological conditions (related to the cost structure of small firms) and behavioral conditions (related to the formation of expectations). Both these factors, introduce a problem of credibility of the dominant firm, which could invalidate the viability of the equilibrium in the long run.

In order to address, at least partially, this problem, we introduce a more complex model in which, in each period, all firms in the competitive fringe have the opportunity to change the heuristics with which they form expectations, based on an assessment based on past performance (see e.g [8]). A certain level of inertia to behavior change affects the probability with which the expectation formation mechanisms are updated and, consequently, the speed of the entire adjustment process. We analyze this framework by means of numerical simulations, which suggest that the lower the announced price, the higher the average market price and also its volatility. Consequently, the optimal price will depend on the risk attitude of the dominant firm. If the dominant firm completely disregards its profit volatility and is only interested in maximizing its average profit, then the optimal choice is the same as in the basic version of the model: to announce as low a price as possible. However, if the dominant firm is characterized by a certain degree of risk aversion, then the optimal choice could imply a radical change in strategy, leading to announce a much higher reference price in order to reduce profit volatility.[4]

Indeed, by calculating the utility of the dominant firm for various degrees of risk aversion (using an iso-elastic CRRA utility function), we find that the higher the degree of risk aversion, the higher the announced optimal price. Finally, we check the robustness of these results with respect to the value of the inertia parameter and find that they qualitatively hold, although with possibly different nuances.

2 The baseline model

We present here a model of a market with one dominant firm and a competitive fringe along the lines sketched above.

Let's consider the market for a good with linear demand function

$$p = a - bq \tag{1}$$

$(a, b > 0)$ which we take as given. The supply side of the market is populated by two different types of actors: *(i)* one big, rational, profit maximizing firm, and *(ii)* a given pool of small, naive, price taking firms.
The big firm strategically announces a price target, p^*, which influences small firms' behavior, in order to maximize profits in the long-run. We assume that such reference price is announced once and remains the same over the entire time horizon: there is no possibility (due to some credibility issue) to change such value strategically period after period. We also assume that there is no entry-exit possibility.

Small firms behavior. Small firms belonging to the competitive fringe are assumed to be price-taking. We depart from the standard literature on this subject by considering a pool of small firms having heterogeneous costs and expectations. In particular:
(1) Total costs are quadratic, and depending on a positive parameter, γ^i, which shapes the efficiency of a firm at the individual level

$$CT^i = \frac{\left(q^i\right)^2}{2\gamma^i} + F_i$$

(2) Price expectation of firm i is obtained by a convex combination of the announced reference price p^*, and of the last observed price p_t

$$p^i_{t+1} = p^* + \alpha^i \left(p_t - p^*\right)$$

with $0 \leq \alpha^i \leq 1$. In this way, each firm expectations balance the reference (taken as a kind of *fundamental value*) together with its present deviation from the actual price. This includes the special extreme cases of *static fundamental expectations* (for $\alpha = 0$), and of *myopic expectations* (for $\alpha = 1$).
At each point in time t, a firm i produces a quantity q^i_t such that price expectation and marginal cost are equal

$$\frac{q^i_t}{\gamma^i} = p^i_t \Rightarrow q^i_t = \gamma^i p^i_t$$

corresponding to an expected profit of

$$\Pi^i_t = p^i_t q^i_t - CT^i_t = \frac{\left(q^i_t\right)^2}{2\gamma^i} - F_i$$

Assume there are n small firms in the market, and define $\boldsymbol{\gamma}$ and $\boldsymbol{\alpha}$ as the n-dimensional (column) vectors of all γ^i and α^i, then the total quantity they produce at time $t+1$ is

$$q^S_{t+1} = \boldsymbol{\gamma}' \left(\mathbf{1} - \boldsymbol{\alpha}\right) p^* + \boldsymbol{\gamma}' \boldsymbol{\alpha} p_t$$

where the products $\boldsymbol{\gamma}' \left(\mathbf{1} - \boldsymbol{\alpha}\right)$ and $\boldsymbol{\gamma}'\boldsymbol{\alpha}$ give a measure of the roles played, respectively, by the announced price and by the last realized price in molding firms decisions.

[4] For a firm, different degrees of risk aversion may result from institutional and financial constraints of various kinds. Financial constraints can be the outcome of difficulties in accessing credit. Institutional constraints may instead be the consequence of a specific active policy by the policy maker: for example, as the result of a reduction in the duration of contracts from which public concessions derive, or of the adoption of specific schemes of CEO remuneration (see e.g. [1]).

Dominant firm behavior. The dominant firm can anticipate small firms behavior and acts as a monopolist on the residual demand. Under the assumption of linear costs, $CT = cq^B + F$, at each time period t, profits are given by

$$\Pi_t^B = p_t q_t - CT_t = \left(a - b\left(q_t^B + q_t^S\right)\right) q_t^B - cq_t^B - F$$

so that profit maximization implies

$$a - 2bq_t^B - bq_t^S - c = 0 \Rightarrow q_t^B = \frac{a-c}{2b} - \frac{q_t^S}{2}$$

and

$$\Pi_t^B = (p_t - c)\left(\frac{a-c}{2b} - \frac{q_t^S}{2}\right) - F = (p_t - c)\left(\frac{a-c}{2b} - \frac{\boldsymbol{\gamma}'\left(\mathbf{1}-\boldsymbol{\alpha}\right)p^* + \boldsymbol{\gamma}'\boldsymbol{\alpha}p_{t-1}}{2}\right) - F$$

Dynamics. So far, we have implicitly assumed that each firm sticks to a specific expectation rule (i.e. to a specific value for α) over the whole time horizon. For the time being we maintain this hypothesis, which will be dropped in Section 3. Under this simplifying assumption, the dynamics of the system is described by the following set of equations

$$\begin{cases} p_{t+1}^i = p^* + \alpha^i\left(p_t - p^*\right) \qquad i = 1\ldots n \\ q_{t+1}^i = \gamma^i p_{t+1}^i \qquad i = 1\ldots n \\ q_{t+1}^S = \sum_{i=1}^n q_{t+1}^i \\ q_{t+1}^B = \frac{a-c}{2b} - \frac{q_{t+1}^S}{2} \end{cases} \tag{2}$$

which, given equation (1) and hence $p_{t+1} = a - b(q_{t+1}^B + q_{t+1}^S)$, determine the following law of motion

$$p_{t+1} = \frac{a+c}{2} - \frac{b}{2}\left(\boldsymbol{\gamma}'\left(\mathbf{1}-\boldsymbol{\alpha}\right)p^* + \boldsymbol{\gamma}'\boldsymbol{\alpha}p_t\right) \tag{3}$$

The dynamics is linear and its unique steady state is $\hat{p}$

$$\hat{p} = \frac{a + c - b\boldsymbol{\gamma}'\left(\mathbf{1}-\boldsymbol{\alpha}\right)p^*}{2 + b\boldsymbol{\gamma}'\boldsymbol{\alpha}} \tag{4}$$

which depends on several parameters and, in particular, on the size n and types γ^i, α^i of the competitive fringe. Observe that the equilibrium $\hat{p}$ is decreasing in p^*. The intuition is that if the reference price is high, small firms will adapt their expectations to it and hence increase production, thus leading to a price decline.

Credibility of the announcement. In general, the announced price p^* will not be consistent with the steady state, in the sense that $p^* \neq \hat{p}$, so the dominant firm will need to consider the effect of the chosen reference price on the value of the steady state that will be actually achieved as well as on its stability. Given the structure of market demand and the number n of small firms, the steady state will be coherent with the reference price itself if and only if the announced price p^* is[5]

$$p^* = \frac{a+c}{2+b\Gamma} \qquad \textit{with} \qquad \Gamma = \sum_{i=1}^n \gamma^i$$

Stability conditions. The map is linear in p_t, hence stability conditions

$$-1 < \left.\frac{dp_{t+1}}{dp_t}\right|_{p_t = p^*} < 1$$

are satisfied if and only if[6] $\boldsymbol{\gamma}'\boldsymbol{\alpha} < 2/b$, which we can also write as

$$\frac{\boldsymbol{\gamma}'}{\Gamma}\boldsymbol{\alpha} < \frac{2}{b\Gamma} \tag{5}$$

[5]This is the equilibrium price considered in the standard (textbook) static model of a dominant firm with a competitive fringe.

[6]This is an immediate consequence of

$$\frac{d\left(\frac{a+c}{2} - \frac{b}{2}\left(\boldsymbol{\gamma}'\left(\mathbf{1}-\boldsymbol{\alpha}\right)p^* + \boldsymbol{\gamma}'\boldsymbol{\alpha}p_t\right)\right)}{dp_t} = -\frac{b\boldsymbol{\gamma}'\boldsymbol{\alpha}}{2} < 0$$

showing that stability is granted if the weighted average of the behavioral parameters α^i with weights equal to $\frac{\gamma^i}{\Gamma}$, representing a measure of the fraction of small-firms total quantity produced by firm i, is less than $\frac{2}{b\Gamma}$. Although we assume here that the number n of small firms is fixed, it is easy to see that this condition loosens as n increases. In particular, as the number of small firms grows, the stability condition tends to[7]

$$E\left[\alpha^i\right] < \frac{2}{b\hat{\Gamma}} \qquad with \quad \hat{\Gamma} = \lim_{n\to+\infty} \Gamma$$

Constraints. In order for the dynamics to make sense from the standpoint of its economic interpretation, a few constraints must be satisfied.

A first trivial condition is that prices be non-negative. However, a tighter condition we need, is that prices should never fall below c, because otherwise there would be no room left for the dominant firm. This is granted if the overall quantity produced by the small firms is smaller than the competitive one

$$a - b\sum_{i=1}^{n} \gamma^i p^i > c \Leftrightarrow \sum_{i=1}^{n} \gamma^i p^i < \frac{a-c}{b} \tag{6}$$

This condition being met is not sufficient to grant positive profits for all firms, unless fixed costs are all zero. In order to have an equilibrium sustainable in the long run, profits must be non-negative for all firms in the market at the steady state $\hat{p}$: for small firms, letting $\hat{p}^i = p^* + \alpha_i\left(\hat{p} - p^*\right)$, it must be

$$\Pi_i\left(\hat{p}^i\right) = \gamma^i \hat{p}^i \left(\hat{p} - \frac{\hat{p}^i}{2}\right) - F^i = \frac{\left(\hat{q}^i\right)^2}{2\gamma^i} - F_i \geq 0 \Leftrightarrow \frac{\gamma^i\left(\hat{p}^i\right)^2}{2} - F_i \geq 0 \Leftrightarrow \gamma^i \geq \frac{2F_i}{\left(\hat{p}^i\right)^2}$$

while for the dominant firm the condition is

$$\Pi\left(\hat{p}\right) = \left(\hat{p} - c\right)\left(\frac{a-c}{2b} - \frac{q^S}{2}\right) - F \geq 0$$

which is satisfied if condition in (6) holds and F is small enough.

In general we will suppose that the dominant firm will be sufficiently efficient to attain positive profits for every possible choice of Γ (hence, for every possible small-firms' production level).

2.1 Optimal choice of the reference price

We now turn to the problem of the strategic choice of the announced reference price p^*, whose value we assume the dominant firm chooses with the goal of profit maximization. To this end there can be different approaches: taking into account only the long-term outcome or considering the entire inter-temporal stream of profits. We assume that the stability conditions stated in (5) are satisfied (hence that the average long-run profit is well approximated by that attainable on the steady state) and we focus here on the simpler case, while leaving the more complex task to the later Section 3.

Let's consider a dominant firm aiming at maximizing the long-term economic outcome. If the announced price has to be consistent (e.g., for some technical or institutional reason) with the equilibrium, then, as we have seen, the only possible choice is to set $p^* = \frac{a+c}{2+b\Gamma}$, corresponding to a long-term average profit equal to $\Pi = \frac{(p-c)(a-c-b\Gamma p)}{2b} - F$. On the contrary, if it is possible for the dominant firm to choose the reference price regardless of the long-run credibility issue discussed above, then by imposing first order conditions on

$$\max_{p^*} \Pi = \max_{p^*} \left(\hat{p} - c\right)\left(\frac{a-c}{2b} - \frac{q^S}{2}\right) - F = \max_{p^*} \frac{\left(\hat{p} - c\right)}{2}\left(\frac{a-c}{b} - \boldsymbol{\gamma}'\left(\mathbf{1} - \boldsymbol{\alpha}\right)p^* - \boldsymbol{\gamma}'\boldsymbol{\alpha}\hat{p}\right) - F$$

we get[8]

$$p^* = \frac{a - c - bc\boldsymbol{\gamma}'\boldsymbol{\alpha}}{b\boldsymbol{\gamma}'\left(\mathbf{1} - \boldsymbol{\alpha}\right)}$$

[7] This follows from the Kolmogorov's strong law of large numbers. See [12] for more details.

[8] From

$$\begin{aligned} \frac{d\Pi}{dp^*} &= \frac{\partial \Pi}{\partial p^*} + \frac{\partial \Pi}{\partial \hat{p}}\frac{\partial \hat{p}}{\partial p^*} = -\frac{\left(\hat{p} - c\right)}{2}\boldsymbol{\gamma}'\left(\mathbf{1} - \boldsymbol{\alpha}\right) + \left(\frac{a-c}{2b} - \frac{\boldsymbol{\gamma}'\left(\mathbf{1} - \boldsymbol{\alpha}\right)}{2}p^* + \left(c - 2\hat{p}\right)\frac{\boldsymbol{\gamma}'\boldsymbol{\alpha}}{2}\right)\left(-\frac{b\boldsymbol{\gamma}'\left(\mathbf{1} - \boldsymbol{\alpha}\right)}{2 + b\boldsymbol{\gamma}'\boldsymbol{\alpha}}\right) = \\ &= \frac{2\boldsymbol{\gamma}'\left(\mathbf{1} - \boldsymbol{\alpha}\right)}{\left(2 + b\boldsymbol{\gamma}'\boldsymbol{\alpha}\right)^2}\left(c - a + bc\boldsymbol{\gamma}'\boldsymbol{\alpha} + b\boldsymbol{\gamma}'\left(\mathbf{1} - \boldsymbol{\alpha}\right)p^*\right) \stackrel{!}{=} 0 \end{aligned}$$

with the second order derivative being

$$\frac{2b\left(\boldsymbol{\gamma}'\left(\mathbf{1}-\boldsymbol{\alpha}\right)\right)^2}{\left(2+b\boldsymbol{\gamma}'\boldsymbol{\alpha}\right)^2}>0$$

which implies that this solution is a local minimum. Indeed for this value of p^* we have $\hat{p}=c$, hence in this case the small firms would produce the competitive quantity, which however we have ruled out with condition (6), and profits for the dominant firm would be negative $\Pi=-F$. Given that by increasing the announced price p^* the equilibrium price $\hat{p}$ would reduce, and the profits for the dominant firm would remain negative, the solution to the problem is found by reducing p^* to its smallest admissible value. Given that p^* is just an announced price, the only constraints to be satisfied are those related to credibility issues. A trivial lower bound could be $p^*=0$ which is admittedly harder to be taken seriously also by very naive small firms. So, by assuming that the small firms know (to some extent) the marginal cost of the dominant firm, we could set the lower bound at $p^*=c$. For this value of p^* we get

$$\hat{p}_{p^*=c}=\frac{a+c-bc\boldsymbol{\gamma}'\left(\mathbf{1}-\boldsymbol{\alpha}\right)}{2+b\boldsymbol{\gamma}'\boldsymbol{\alpha}}$$

and we know for sure (out of convexity of the profit function) that $\Pi_{p^*=c}>\Pi_{p^*>c}$.

Summarizing, for the dominant firm pursuing the goal of profit maximization, the choice of reference price to announce should fall on the lowest possible value; however, this choice can be made difficult by the need to maintain the credibility of the announcement itself, especially in the long run. Moreover, the above considerations implicitly assume (at least) local stability (but global stability would be more appropriate) of the steady state. We have seen earlier in (5), that this is guaranteed either if the total output of small firms is sufficiently large (i.e., Γ is large) or if expectations are strongly influenced by the announced price (i.e., α^i are small). If this were not the case, long-run profits would no longer be those obtainable in the steady state and it would be necessary to switch to an out-of-equilibrium analysis. Finally, credibility and stability issues are also intertwined with the fact that expectations are not confirmed in equilibrium if $p^*\neq\hat{p}$ unless $\alpha^i=1$. All of these concerns will be addressed in the following Section 3.

3 Expectation dynamics and learning

We have seen that in the basic setting discussed above the dominant firm would be willing to announce a very low reference price p^*, under the assumption that the credibility of such an announcement will not be jeopardized in the long run by the difference between p^* and $\hat{p}$. We now assume that small firms adapt their expectations' rule to the environment they face: in particular we posit that a persistent difference between p^* and p_t ceases to be sustainable on the steady state $\hat{p}$, causing the simple mechanism that led to the optimal choice of the dominant firm to break down.

We consider an evolutionary mechanism as a device to implement the choice of the predictor by each firm, which therefore ceases to be exogenous to the model. The work in [8] and the related branch of literature about heuristic switching largely inspire the exercise we conduct in this section.

We have n firms and m predictors consisting each in a different kind of value for the adaptive parameter alpha. In period t each firm assesses the "fitness" of each predictor based on its relative performance in terms of forecasting error of the previous period, given the last observation for the price and the firm's prediction at time $t-1$. Each small firm considers whether to switch to a new predictor, randomly selected among the available predictors, with probabilities that reflect the fitness measure just described: such probabilities are computed by means of the discrete choice model as in [8], using a given intensity of choice parameter β, measuring how sensitive the firms are to the predictors past performance. Formally, for a firm, the probability of choosing predictor j at time t is

$$\eta_t^j=\frac{e^{-\beta\left(p_t^j-p_t\right)^2}}{\sum_{j=1}^k e^{-\beta\left(p_t^j-p_t\right)^2}} \tag{7}$$

where $p_t^j=p^*+\alpha^j\left(p_{t-1}-p^*\right)$ is the price forecast generated by predictor j, using reference price and price from the previous period. Each small firm however resists change with some degree of inertia[9] and therefore

[9]The existence of inertia in expectations, implying that not all the participants revise their rule in every period is suggested by several authors. For example, [23] show theoretically that forward looking expectations may imply inertia. Such kind of behavior is also widely reported in experiments (see e.g. [19]. Also, several papers introduce inertia within the heuristic switching mechanism with the same logic followed here (see e.g. [4, 10, 11, 13])

actually switches to the new predictor selected as described, with a probability equal to $1-\delta$ where $\delta \in [0,1]$ is the "inertia" parameter. The remaining structure of the above model is unchanged and the simulations carried out below implement such structure numerically.[10]

In order to compare this case with the previous one based on static expectations one possibility is to reduce the model to a deterministic version by slightly twisting the model in the following way. We can assume that, for each i, γ^i defines the production of a continuum of firms (instead of the production of a single firm) and that these firms act on the basis of expectations defined by a certain probability distribution on α^j (rather than all γ^i-types using the same value of alpha) determined by the η^j in (7). In this case, letting $\boldsymbol{\eta}$ and $\boldsymbol{p^e}$ the column vectors of probabilities and of expectations, the production of the small firms can be written as

$$q^S = \sum_{i=1}^{n} q^i = \sum_{i=1}^{n} \gamma^i \sum_{j=1}^{m} \eta^j p^j = \sum_{i=1}^{n} \gamma^i \boldsymbol{\eta}' \boldsymbol{p^e} = \Gamma \boldsymbol{\eta}' \boldsymbol{p^e}$$

or, more explicitly

$$q^S = \Gamma \boldsymbol{\eta}' \left(\boldsymbol{\alpha} p + (\mathbf{1} - \boldsymbol{\alpha}) p^* \right)$$

Hence, by substituting this equation to the corresponding one in the system (2), and together with (7) we get the dynamical system

$$\begin{cases} p_{t+1} = \frac{a+c}{2} - \frac{b}{2} \Gamma \boldsymbol{\eta}_t' \left(\boldsymbol{\alpha} p_t + (\mathbf{1} - \boldsymbol{\alpha}) p^* \right) \\ \eta_{t+1}^j = \frac{e^{-\beta (p^* + \alpha_j (p_t - p^*) - p_{t+1})^2}}{\sum_{j=1}^{k} e^{-\beta (p^* + \alpha_j (p_t - p^*) - p_{t+1})^2}} \qquad j = 1 \dots m \end{cases} \tag{8}$$

We show in the following proposition that system (8) has a unique steady state. Further we also argue that given restrictions on the parameters coming from the model, such steady state also lies on the feasible set for the price variable.

Proposition 1 *System* (8) *has a unique steady state. Further, if p^* is not too large, the steady state occurs for a positive price.*

Proof. Observing that the η^j's do not depend on their past value, we can rewrite the system as a second-order difference equation as follows

$$p_{t+1} = \frac{a+c}{2} - \frac{b}{2} \Gamma \frac{\sum_{j=1}^{m} e^{-\beta (p^* + \alpha_j (p_{t-1} - p^*) - p_t)^2} (\alpha_j p_t + (1 - \alpha_j) p^*)}{\sum_{j=1}^{m} e^{-\beta (p^* + \alpha_j (p_{t-1} - p^*) - p_t)^2}} \equiv F(p_t, p_{t-1}) \tag{9}$$

Our strategy to show that the equation has a single steady state is to study the function F on the restriction $p_t = p_{t-1}$ and to search for its fixed points. Indeed fixed points of (9) correspond to fixed points of the function $f(p) \equiv F(p,p)$. To ease the notation slightly we study the fixed points $p - p^* = f(p - p^*)$ and hence, setting $x = p - p^*$, the fixed points of the following function

$$g(x) = \frac{a+c}{2} - \frac{b}{2} \Gamma \frac{\sum_{j=1}^{m} e^{-\beta x^2 (1-\alpha_j)^2} (\alpha_j x + p^*)}{\sum_{j=1}^{m} e^{-\beta x^2 (1-\alpha_j)^2}} - p^* \tag{10}$$

To this end we simply show that $g(x)$ is stricly decreasing. Indeed:

$$\frac{dg(x)}{dx} = -\frac{b}{2} \Gamma \frac{\sum_{i=1}^{n} \sum_{j>i} \left(2x^2 \beta (2 - \alpha_i - \alpha_j)(\alpha_i - \alpha_j)^2 + \alpha_i + \alpha_j \right) e^{-x^2 \beta \left((1-\alpha_i)^2 + (1-\alpha_j)^2 \right)} + \sum_{i=1}^{n} \alpha_i e^{-2x^2 \beta (1-\alpha_i)^2}}{\left(\sum_{i=1}^{n} e^{-\beta ((1-\alpha_i) x)^2} \right)^2} \tag{11}$$

[10] Observe that the considered set of predictors require adding a fraction of the difference between actual realized price and the reference price to the reference price itself, while here firms are assumed to choose among different predictors with probabilities matching the relative precision of the predictors at the antecedent date. Hence, firm types can be considered as a whole as in the previous sections, since using the same alphas implies generating the same point prediction. Probabilities and fractions can therefore be considered the same in [8], where such possibility (arising from the nature of the predictors considered) permits to analyze the model in an entirely deterministic way.

which, since the α's are in the unit interval, is negative. The way to show that (11) is indeed $g'(x)$ is by observing that, letting $N_i = e^{-\beta x^2(1-\alpha_i)^2}(\alpha_i x + p^*)$ and $D_j = e^{-\beta x^2(1-\alpha_j)^2}$ we get that the numerator of the derivative is the sum for each couple (i, j) of the terms

$$\frac{dN_i}{dx}D_j - N_i\frac{dD_j}{dx} = \left(2x\beta\left(2-\alpha_i-\alpha_j\right)\left(\alpha_i-\alpha_j\right)\left(p^*+x\alpha_i\right)+\alpha_i\right)e^{-x^2\beta\left((1-\alpha_i)^2+(1-\alpha_j)^2\right)}$$

Observing that for $i \neq j$

$$\frac{dN_i}{dx}D_j - N_i\frac{dD_j}{dx} + \frac{dN_j}{dx}D_i - N_j\frac{dD_i}{dx} = \left(2x^2\beta\left(2-\alpha_i-\alpha_j\right)\left(\alpha_i-\alpha_j\right)^2+\alpha_i+\alpha_j\right)e^{-x^2\beta\left((1-\alpha_i)^2+(1-\alpha_j)^2\right)}$$

and that

$$\frac{dN_i}{dx}D_i - N_i\frac{dD_i}{dx} = \alpha_i e^{-2x^2\beta(\alpha_i-1)^2}$$

the result immediately follows.

Finally observe that

$$f(p^*) = g(0) + p^* > 0 \iff g(0) = \frac{a+c}{2} - \frac{b}{2}\Gamma p^* - p^* > -p^*$$

whenever $p^* < \frac{a+c}{b\Gamma}$. In turn, since $f' = g' < 0$, the fixed point lies in the first quadrant. ■

A detailed analytical analysis of this model is beyond the scope of this paper. However some insights regarding the qualitative aspects of the local dynamics of the system emerge upon closer inspection. In particular, expectations are confirmed in equilibrium if (and only if) $\alpha^j = 1$. In fact, from (8) immediately follows that $\alpha^j = 1$ is the most frequently used predictor in equilibrium, and that such probability grows with the distance between p^* and the steady state. High values of α are therefore likely to emerge on the steady state, and this is a possible determinant of instability of the steady state (see the discussion in the previous section). Also, a second source of instability comes from the interdependence of p_t and η_t^j which our simulations suggests plays a role in determining the long-run outcome of the dynamics.

3.1 Simulations

In this section, we report the results of simulations of the model to study the relationship between the optimal choice of the dominant firm and some of the parameters involved.

To carry out the simulations we consider the model described in (2) where the demand parameters are set to $a = 1000$ and $b = 2$, and the dominant firm marginal cost is $c = 0$. We also consider a fixed number, 50, of small firms having (inverse) marginal costs described by the equispaced vector $\boldsymbol{\gamma}$ with $\gamma^i = \frac{2i}{n(n+1)}$, whose terms sum up to 1. The set of predictors shall be sufficiently large to represent fairly well the possible types, from $\alpha = 0$ that mimics static fundamental expectations to $\alpha = 1$ corresponding to naive (myopic) expectations. We took such number to be equal 11,[11] so that the alphas among which to choose were 0, 0.1, ..., 0.9, 1. The intensity of choice in the heuristic switching mechanism (7) is $\beta = 0.001$, while inertia, determining the probability to change the predictor, is set to $\delta = 0.75$. All fixed costs are assumed to be zero.

The basic simulation runs for T time periods[12] and uses a given p^*. A variety of dynamic behaviors can be observed, depending on p^*, as can be seen by inspecting Figure 1.

[11]Increasing this value did not impact significantly on the results. Being significantly more parsimonious (e.g. two or three different alphas) instead imposed us to make arbitrary choices about the actual values chosen for the different alphas and results were overly sensitive to such arbitrary choices.

[12]The simulations were carried out considering a time span of length $T = 500$.

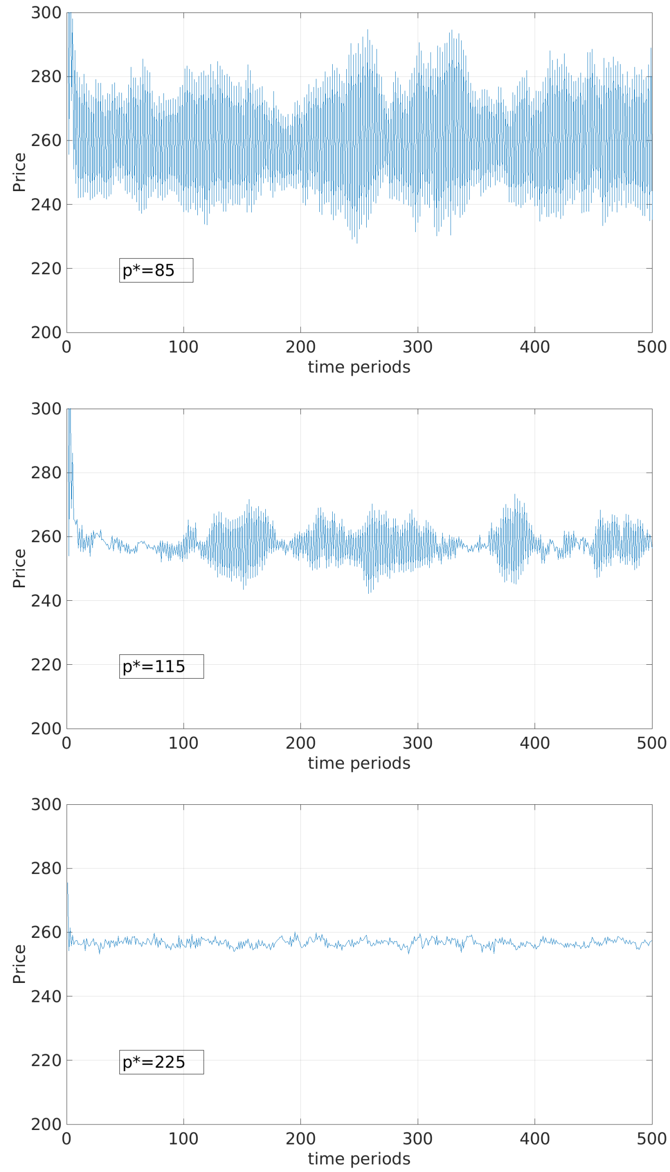

Figure 1: Examples of price dynamics for different values of p^*

The dynamics of price heavily depends on the announced p^*. The more p^* is near the self-confirming value $\hat{p}$ the more the fluctuations - and hence uncertainty - fade. Figure 1 shows that, as in the baseline model, a lower announced price implies a lower supply from the competitive firms and a higher profit, on average, for the dominant firm. However, in this case the realized market price is far from the announced price and for this reason a significant fraction of the firms place a lower weight on the p^* in the following period. As a result, the fringe will increase their production and the market price will decrease, moving towards p^*. In turn, this fact will push the fringe to reduce their production, thus starting the fluctuations that characterize the depicted time series. On the contrary, when the announced price is high, closer to the self-confirming value, the discrepancy between the announced and the realized price will be low and the firms of the fringe will only marginally adjust their expectations, hence reducing volatility.

This fact has direct consequences over the stream of profits which the dominant firm could expect as a consequence of its announcement. Figure 2 shows six examples related to as many different values of p^*.

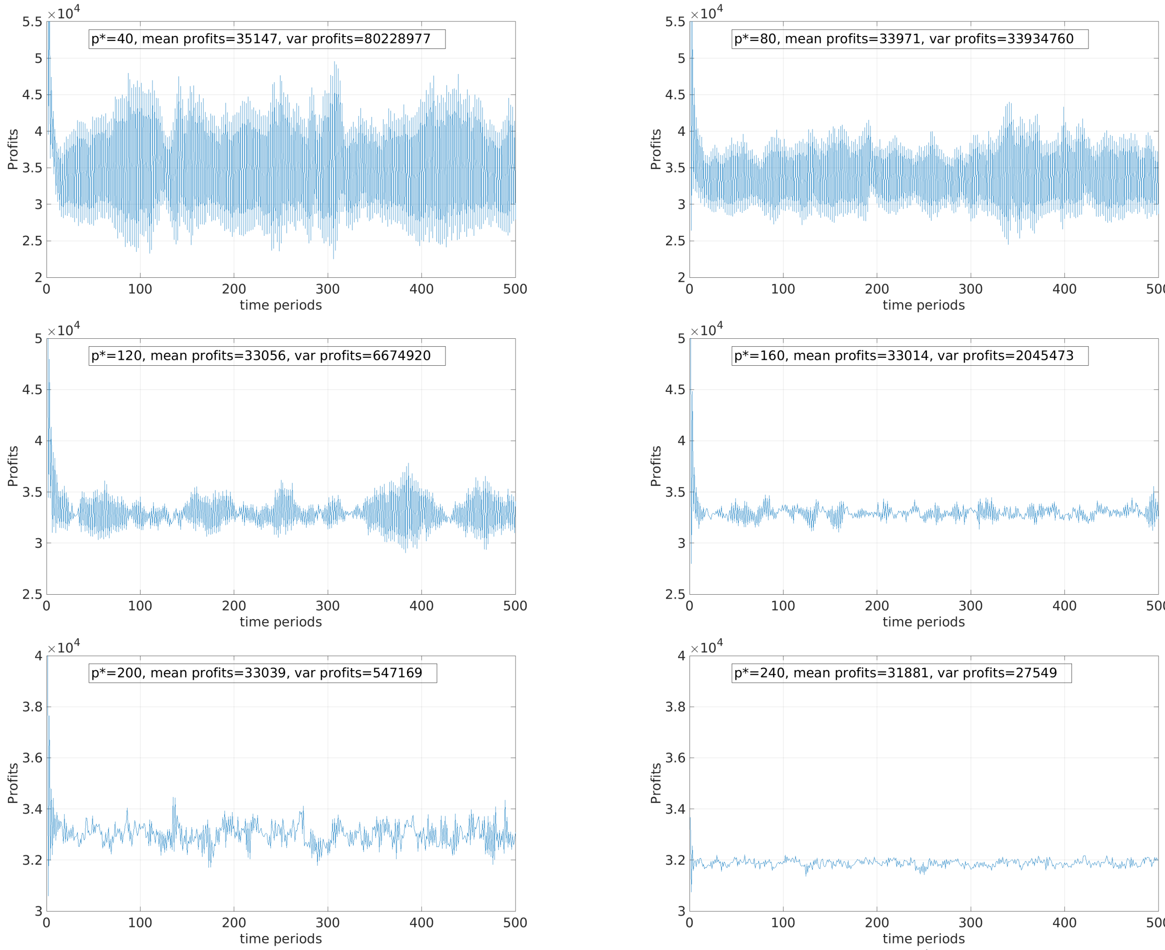

Figure 2: Examples of profit dynamics for different values of p^*

Figure 2 suggests that the choice of p^* is not trivial, if the dominant firm needs to consider the stability of its revenues. Indeed a Monte-Carlo simulation in which the basic exercise is repeated, and mean profits and their variance are recorded, helps underline that there is indeed a non-obvious choice to be done in terms of mean-variance combination of the expected profits as a function of the announced reference price p^*. Figure 3 has the results.

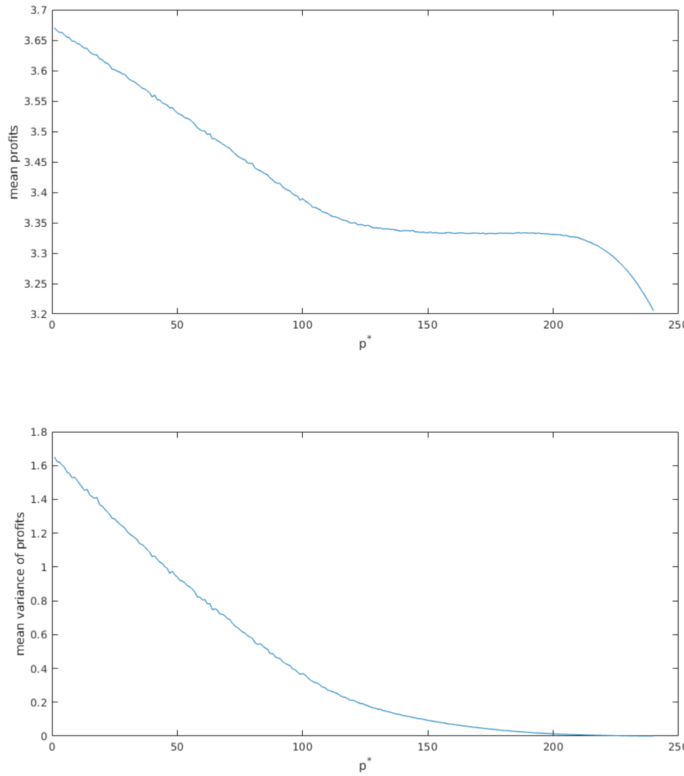

Figure 3: Dominant firm's mean profits and their variance as functions of the announced reference price

In order to take into account both elements, we explored the incentives for a dominant firm deriving utility from profits by way of a Constant-Relative-Risk-Aversion iso-elastic utility function

$$U(\pi) = \frac{\pi^{1-\rho} - 1}{1-\rho}$$

where a larger value of the relative risk aversion coefficient ρ implies a higher curvature of the utility function and hence a larger discount of variability.[13]

In figure (4), the average utility is shown, for different values of p^* as ρ increases. Firstly, as risk aversion increases, average utility significantly decreases. On closer inspection, we note the following:

- $\rho = 1.2$: the maximum value of average utility is obtained for low values of p^*. The utility decreases to $p^* = 100$ and then increases slightly and reaches a (local) maximum around 200;
- $\rho = 1.6$: the average utility is bimodal, it has two maxima and one minimum (always around $p^* = 100$);
- $\rho = 2$: the average utility increases as p^* increases and has a maximum around 200.

[13]The empirical literature on this subject, see e.g. [27], identifies the interval between 1 and 2 as the most representative set of values for the coefficient of relative risk aversion in the case of individual decisions. This range of values continues to be sufficiently wide and significant also in our context, although here risk aversion may be more the result of possible financial constraints of the company than of psychological attitudes towards risk.

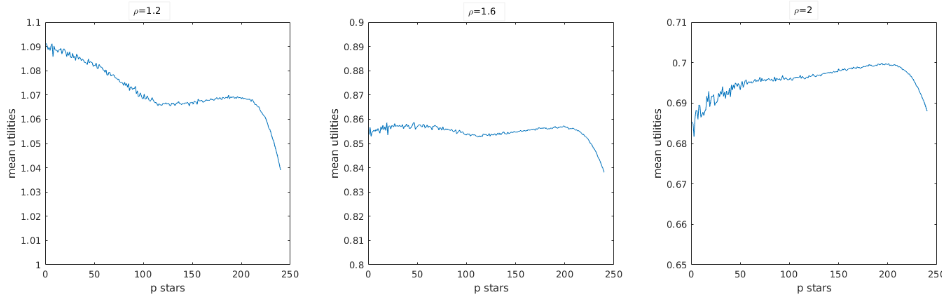

Figure 4: Mean Utilities for different risk aversion

Note that (see figure 3) for sufficiently low value of the degree of risk aversion the utility has a local minimum around 100 because up to that value the reduction of the average profit is not compensated by the reduction in its variance. However, after 100 the expected profit remains almost constant up to around 200, while the variance is always substantially decreasing and for these reasons the expected utility is increasing in such interval. For higher values of the degree of risk aversion the average utility became monotonically increasing up to more than 200.

It seems to follow from these observations that the optimal value of the reference price for the dominant firm is an increasing function of risk aversion. To fully investigate the issue we examined, through Monte-Carlo simulation, the best average choice of p^* in terms of implied mean utility of the stream of profits over T periods, for values of ρ in the range between 1 and 2. These simulations are summarized in Figure 5, which suggests that there can be expected three different regimes as a function of the risk aversion parameter, with a roughly S-shaped growing relationship between risk aversion and optimal reference price p^*. For a low risk aversion, the optimal p^* is lower then 10, and it remains low up to $\rho \approx 1.35$. Then it grows slowly until a risk aversion close to 1.60, after which there is an clear *change of regime*: the optimal value shifts in the upper part of the choice set, around 200.

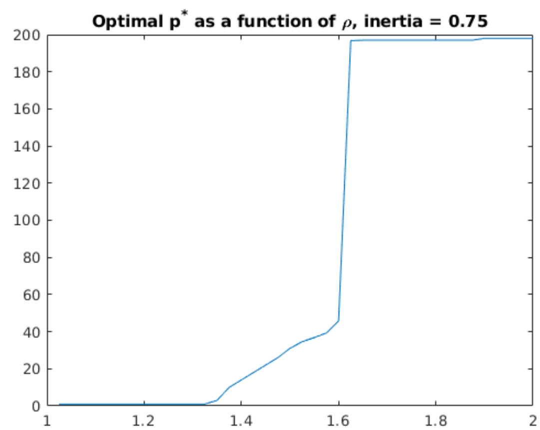

Figure 5: Optimal announced reference price price as a function of risk aversion

The results in figure 5 can be explained by looking at figure 3: for sufficiently low values of the degree of risk aversion the dominant firm prefers to announce a low price because with higher prices the reduction of its average profits are not compensated by the reduction of their variance. This result holds even though for announced prices between 150 and 200 the average profit remains almost constant while the variance continue to considerably diminish (hence suggesting that values of p^* near 200 could be optimal). Conversely, if the degree of risk aversion is past a certain threshold, than the reduction of the variance of the profits is the main objective for the dominant firm and for this reason the optimal announced price moves to around 200.

The results presented so far have focused only on the risk aversion parameter. However, there are various effects in the model that are worth studying: in particular we now focus on the effect of a change in the level of inertia. The reason for doing so is related to the fact that lower inertia should, in principle, increase the responsiveness of small firms to forecast errors with a potential cascading impact on the variance of the large firm's profits, and hence its strategic choices.

In figure (6), we report the mean profits and their variance for increasing values of inertia. It is worth noting that the decreasing relationships between the announced reference price and both the mean profits and their variance seem to hold in general, reproducing the choice dilemma for the dominant firm. However, an unexpected effect of the increase of inertia is that mean profits decrease for all values of p^* and, for very high levels of inertia (when few firms change predictors), the profit curve flattens out. At the same time, the higher the inertia, the smaller the variance in profits. Actually, the lower the value of the inertia parameter, the more similar is this context to the baseline model (where no firms adjust its expectation algorithm from one period to the next). Therefore, these figures prove that the mechanism of expectations adjustment is beneficial for the dominant firm in terms of mean profits. At the same time, such mechanism increases notably the volatility of such profits, entailing considerably oscillations in the market price from one period to the next.

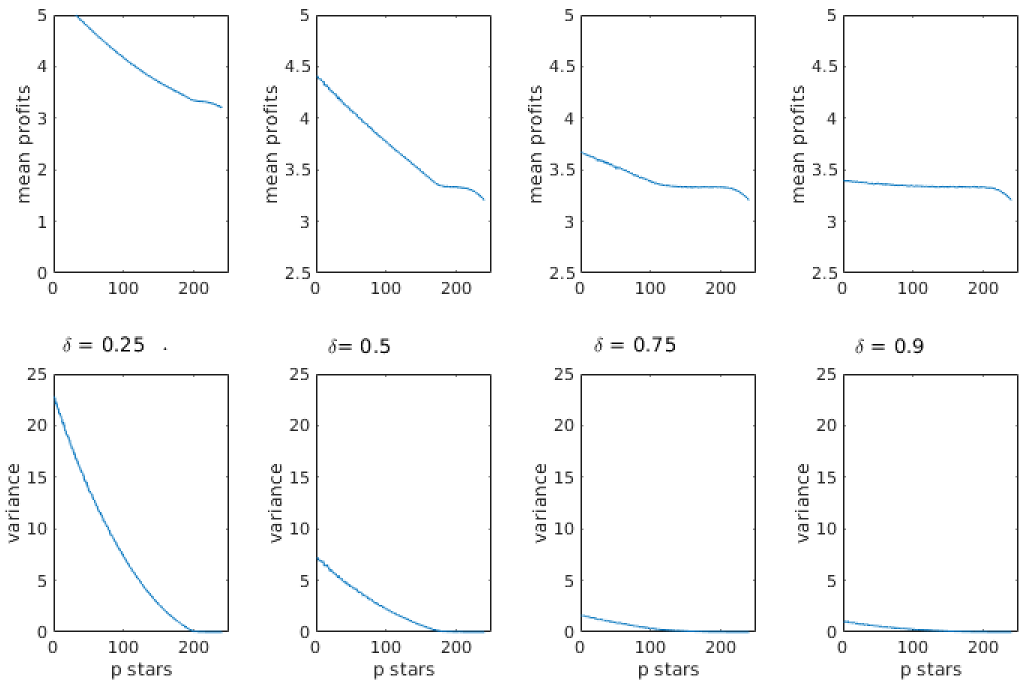

Figure 6: Mean Profits and Variance with different values of inertia

To see if the result obtained for $\delta = 0.75$ and represented in figure 5, of an increasing relation between risk aversion and optimal reference price, is robust we ran again a series of Monte-Carlo simulations for $\delta \in \{0.25, 0.4, 0.5, 0.6, 0.8, 0.9\}$.

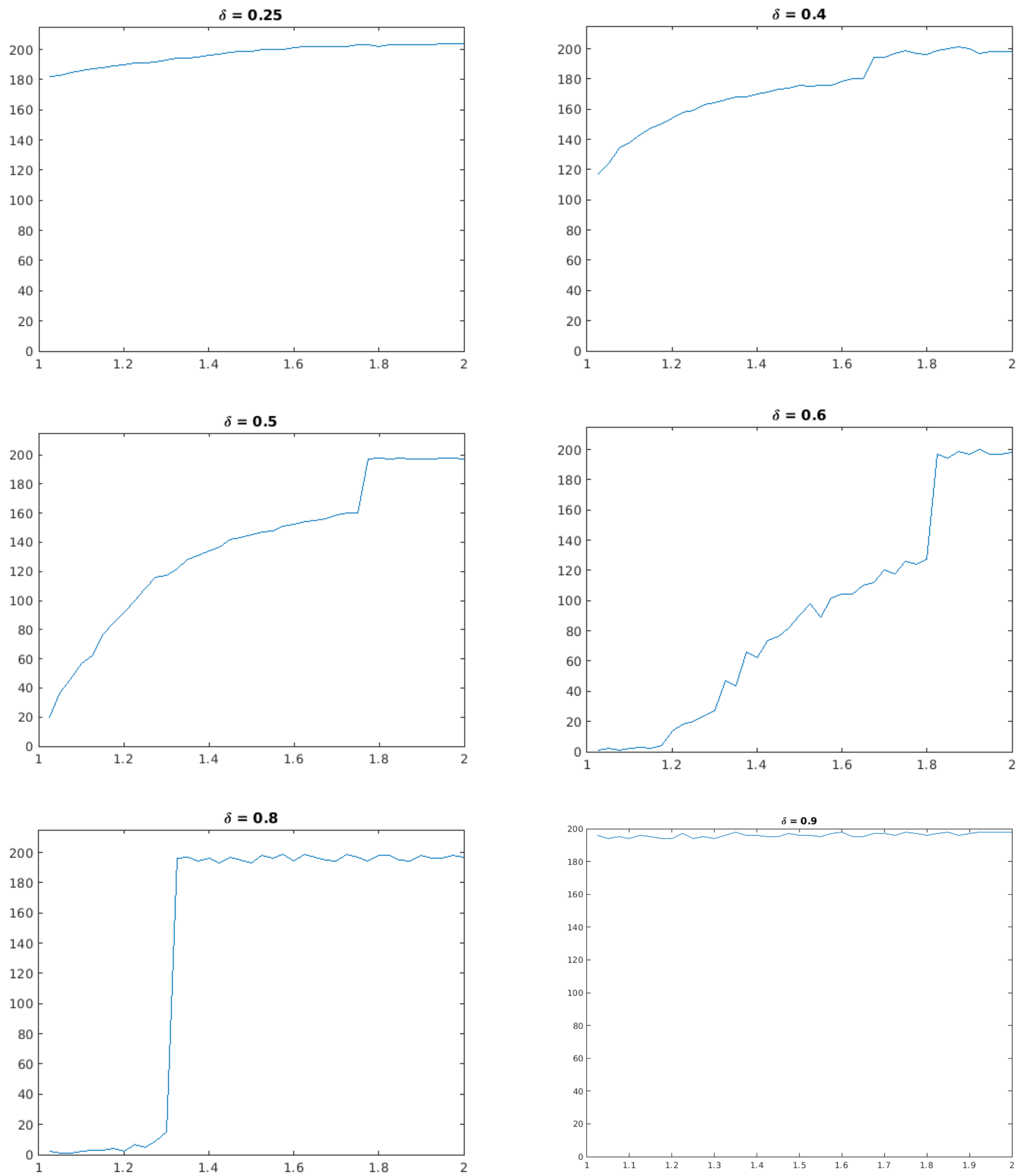

Figure 7: Different values for inertia

Figure 7 shows that the result depicted in Figure 5 - a S-shaped relationship - is robust to intermediate values of the inertia parameter, whereas extremely low or high value seem to compromise it.[14]

[14] We still need to investigate whether this is a by-product of a possible numerical issue or there is a substantial feature of the model.

4 Conclusions

In this paper we study a market in which a dominant firm interacts with a pool of small, naive and price taking firms. The former, in particular, announces the reference price for all periods and maximizes profits, anticipating the behavior of the fringe and behaving, on the residual demand, as a monopolist. Small firms, on the other hand, employ an evolutionary mechanism for their expectations.

Interesting dynamics arise, in which various forces/parameters play a role. The mean utility arising for the dominant firm, given its risk attitude, depends on the particular choice of the reference price to be announced, and, for intermediate values of the risk aversion (in a CRRA utility context), typically follows a bi-modal pattern, with the optimal reference price located at low values or at high values: the upshot is that when the dominant firm can tolerate more unstable return flows it should announce a relatively low price, which in turn will generate higher, but more variable, profits. On the contrary a more risk-averse dominant firm will prefer to announce a higher price whence a less erratic, lower average profit will ensue.

It is worth noting that, somewhat surprisingly, social welfare is increasing in the level of the reference price. Indeed, the higher the reference price, the lower the mean market price and consequently, the lower the social welfare loss. Moreover, from a social welfare standpoint, a higher reference price is preferable also because it entails a lower market price volatility. However, if the dominant firm is interested mainly in its mean profits, disregarding their volatility, then it has an incentive to announce a very low price, inducing higher and more volatile mean prices. In the introduction we emphasized how public service privatization has often given rise to sectors with a dominant firm, the heir of the previous monopolist, and a competitive fringe of small firms. On the basis of our results, policy makers should take particular account of the necessary balance between the potential benefits usually associated with these processes (such as cost reduction, quality improvements, etc.) and the potential risk of higher price volatility and lower consumer surplus. In order to minimize such consequences, authorities should induce the dominant firm to behave as strongly risk averse. This result can possibly be pursued with different instruments, such as specific CEO's remuneration schemes. At the same time, governments should try to mitigate the influence of speculative investors when dominant firms are listed on the stock exchange by choosing solutions similar to the public company format, where the shares of the dominant firms are mainly distributed to small investors who are likely to be moderately interested in average returns and strongly averse to their fluctuations.

There are various directions in which the model could be extended: in particular, there is no guarantee that positive profits are granted to small firms in a given period. Suppose that each firm needs to decide, in each period whether to participate in the market or to withdraw. In this case, the number of firms becomes endogenous, depending on each entry/exit decision and, ultimately, on price expectations and costs. A further extension could regard the maximization of the inter-temporal stream of profits: facing financial constraints and with cash-in-advance problems both the big and the small firms are bound to discount future profits with respect to the present.

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