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Detecting and Measuring Financial Cycles in Heterogeneous Agents Models: An Empirical Analysis

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Detecting and Measuring Financial Cycles in Heterogeneous Agents Models: An Empirical Analysis

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Abstract

This paper proposes a macroeconometric analysis to depict and measure possible financial cycles that emerge due to the dynamic interaction between heterogeneous market participants. We consider 2-type heterogeneous speculative agents: Trend followers tend to follow the price trend while contrarians go against the wind. As agents' beliefs are unobserved variables, we construct a state-space model where heuristics are considered as unobserved state components and from which the conditions for endogenous cycles can be mathematically derived and empirically tested. Further, we specifically measure the length of endogenous financial cycles. The model is estimated using the equity price index for the 1960–2020 period for the UK, France, Germany, and the USA. We find empirical evidence of endogenous financial cycles for all four countries, with the highest frequencies in the USA and the UK.

Key words: Heterogeneous Agent Models, Heterogeneous Expectations, Endogenous Cycles, State Space Model, Period of Cycles

JEL codes: C13, C32, G10, G12, E32

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1 Introduction

For a long time, the rational representative agent paradigm has been a milestone in economic models. This idea influenced the studies of financial markets, based on the fundamental assumption of the efficient market hypothesis [31]. According to this theory, stock prices incorporate the best information about fundamental values and change because of temporary exogenous shocks to the system [11]. Notwithstanding, the rational valuation of an asset has been questioned since the time of John Maynard Keynes, and numerous attacks have been mounted on the dominating homogeneous rational paradigm [23, 35, 24, 34, 30].

In Keynes's view, investors' sentiment and mass psychology (what he called "animal spirits") play a significant role in real financial markets [22]. Waves of optimism and pessimism among agents make the economy more systematically fragile and unstable. In light of this, the observed prices are not governed by an objective view of fundamentals but depend on agents' heuristic beliefs.

Along this line of thinking, heterogeneous agent models (HAMs) question the classical rational agent framework and claim the importance of heterogeneous heuristic expectations. On the theoretical side, HAMs formalize the deviation from rational expectations: a single rational-agent framework is substituted with the idea that economic agents have limited cognitive abilities acting in a market characterized by heterogeneity among individuals [20]. On the empirical side, literature is growing and empirical papers confirm that sentiment dynamics are important in explaining and replicating the observed anomalies in financial markets [29, 36].[1]

The existing HAMs within the empirical literature on equity prices provides a variety of methods for estimation, such as maximum likelihood, nonlinear least square, and moment-based estimations, with no consensus on the estimation strategies ([1, 3, 25, 6, 13, 28, 32, 38, 37]). In recent empirical papers, Thomas Lux [26, 27] uses Markov chain Monte Carlo methods for state-space models. This methodology could be optimal for HAMs because it can distinguish between latent components and observable variables. State-space models have also been used by Gusella and Stockhammer [17] to search for Minskyan financial cycles in asset prices. Gusella and Ricchiuti [18] make steps forward in analyzing cyclical phenomena, assuming that investors have heterogeneous beliefs about the persistence of stock price deviations from the fundamental value. They compare different fundamental benchmarks and estimate the model at various time frequencies.

While the common view is that investor sentiment can affect asset prices, the question about the nature of its effect remains. Underlying this work is a view that agents' behaviors in the market create financial instability as a cyclical endogenous phenomenon.[2] In doing so, we firstly extend the previous models by Gusella and Stockhammer [17], considering two-type heterogeneous speculator agents in a standard small-scale dynamic asset pricing framework. Furthermore, differently from Gusella and Ricchiuti [18], now agents adopt a different speculative bahavior. Speculators, commonly known as chartists, who are subject to self-fulfilling moods that reflect optimistic or pessimistic behavior, form opposite expectations without considering the fundamentals but adapt the forecasting rule as additional price observations become available.

To detect possible endogenous cycles, we rewrite the heterogeneous agents setting in a state-space form. In this way, it is possible to explain an observed variable's behavior by examining the unobserved components' internal dynamic properties. Indeed, eigenvalues analysis can be performed in our discrete dynamic system to study the conditions for oscillations associated with the two unobserved beliefs.

[1] As observed anomalies, we refer to excess volatility, bubbles and crashes, fat-tailed return distributions, and volatility clustering.

[2] We do not concentrate on large-scale HAMs with many agent types but on small-case HAMs restricted to a small group of agent types. For large-scale HAM, also called ABM, see Ref. 9.

In particular, we treat agents' beliefs as unobservable and use the Kalman filter to generate optimal inferences about the unobserved state vector and evaluate whether the conditions for the existence of cycles hold.

In addition, under the idea that cyclical phenomena could be the consequence of the unobserved beliefs of the agents, this paper provides a measure of endogenous financial cycles; extracting the latent components over the state-space model, we can calculate the period and the frequency of cycles due to the heuristic behavior of agents.

Based on these grounds, the paper contributes to the literature in two ways. First, with respect to HAM literature, we confirm and reinforce the view that heterogeneity in agents' characteristics could play a crucial role in shaping the cyclical dynamics of asset prices; the model is kept simple enough to deliver an analytical mathematical solution that can be empirically tested. Second, concerning the endogeneity of financial cycles, the cycle period implied by the transition matrix can be computed, allowing us to map and compare the period cycles from the model with the period cycles predicted by the theory on asset prices. On this regard, whereas most studies have focused on business cycles, few papers have tried to measure the financial cycle and investigate its statistical properties [2]. Existing work argues that financial equity cycles are considered to exist with periods of one up to eight years (short-term periods) while housing prices and debt between eight and thirty years [2].[3]

For the empirical exercise, we examine endogenous cycles in four OECD countries throughout 1960–2020: the United States, Germany, France, and the United Kingdom. This allows us to highlight the similarities and differences between different stock markets and investigate whether the cycles have the same frequency and dynamic persistence.

The rest of the paper is organized as follows. Section 2 discusses the modeling approach and conditions for cycles. Section 3 describes the data and the estimation method. Section 4 summarizes the main findings. Section 5 concludes the paper with final considerations.

2 Methodology

This section develops the methodology to investigate whether and how speculative agents influence asset prices dynamics. Agents are divided into two groups, trend followers (also called momentum traders) and contrarians. These agents observe historical patterns of prices but do not account for the asset's fundamental value to forecast future price movements. Heuristics directly influence agent's expectations and their excess demands, which are reflected in the observed asset price dynamics.

The dynamics of the asset price p_t is determined by the following equation:

$$p_t = p_{t-1} + \omega d_t^m + (1-\omega)\, d_t^c \tag{1}$$

where d_t^m and d_t^c are the weighted (excess) demands of the two different types of agents, trend followers and contrarians respectively. The weights ω and $1-\omega$ are the respective proportions.

The demand functions can be specified as difference between the expected asset price and the current asset price:

$$d_t^m = E_t^m\left(p_{t+1}\right) - p_{t-1} \tag{2}$$

$$d_t^c = E_t^c\left(p_{t+1}\right) - p_{t-1} \tag{3}$$

[3]This is also the range that statistical filters target when seeking to extract the cyclical component.

From Eqs. (2) and (3) we notice that excess demands are strictly related to heterogeneous expectations.

Momentum traders believe that past price movements tend to repeat in the near future. For this reason, trend follower expectations are expressed as

$$E_t^m (p_{t+1}) = p_{t-1} + \beta (p_{t-1} - p_{t-2}) \quad \beta > 0 \tag{4}$$

where β is a positive reaction coefficient. From Eq. (4), when the asset price is above its value at previous time, economic agent optimistically believes in a future price increase. On the opposite, when the asset price is below its value at previous time, economic agent pessimistically believes in a future price decrease. One implication of this is that trend followers speculate based on the extrapolation of observed prices thus buying (selling) the asset when its price has increased (decreased).

On the other hand, contrarians believe that past price movements tend to revert in the near future. Their expectation is thus expressed as:

$$E_t^c (p_{t+1}) = p_{t-1} + \phi (p_{t-1} - p_{t-2}) \quad \phi < 0 \tag{5}$$

where ϕ denotes the parameter which captures the agent's reaction. In the case of boom or bust, contrarians expect market prices to revert to the value of previous time.

If we substitute E_t^m and E_t^c in Eqs. (2) and (3), for trend followers we obtain

$$d_t^m = \beta (p_{t-1} - p_{t-2}) \tag{6}$$

while for contrarians, we have

$$d_t^c = \phi (p_{t-1} - p_{t-2}) \tag{7}$$

As agents' beliefs are unobserved variables, HAM can be formulated in a state-space form, with the observation equation and the unobserved state components. Substituting Eq. (1) in Eq. (6), the excess demand for trend follower can be rewritten as:

$$d_t^m = \beta\omega d_{t-1}^m + \beta (1 - \omega) d_{t-1}^c \tag{8}$$

While substituting Eq. (1) in Eq. (7), the excess demand for contrarians can be rewritten in the following way:

$$d_t^c = \phi\omega d_{t-1}^m + \phi (1 - \omega) d_{t-1}^c \tag{9}$$

We set

$$\begin{cases} a_{11} = \beta\omega \\ a_{12} = \beta (1 - \omega) \\ a_{21} = \phi\omega \\ a_{22} = \phi (1 - \omega) \end{cases} \tag{10}$$

such that, in a stochastic form, we finally obtain:

$$d_t^m = a_{11} d_{t-1}^m + a_{12} d_{t-1}^c + \varphi_t \qquad \varphi_t \sim \mathrm{N}\left(0, \sigma_\varphi^2\right) \tag{11}$$

$$d_t^c = a_{21} d_{t-1}^m + a_{22} d_{t-1}^c + \eta_t \qquad \eta_t \sim \mathrm{N}\left(0, \sigma_\eta^2\right) \tag{12}$$

where φ_t and η_t are the individual disturbance term which are normally distributed with mean zero and variance σ_φ^2 and σ_η^2 respectively.

Eqs. (11) and (12) are the so-called state equations. Together with the observed asset price (Eq. 1), they represent our state-space model. For any state space model, the state equation must be a first-order stochastic difference equation [10]. In matrix-vector representation, the observed equation is:

$$\tilde{p}_t = \begin{pmatrix} \omega & 1-\omega \end{pmatrix} \begin{pmatrix} d_t^m \\ d_t^c \end{pmatrix} \qquad \tilde{p}_t = p_t - p_{t-1}$$

Taking into account Eqs. (11) and (12), we recover the transition equation of the state-space model:

$$\begin{pmatrix} d_t^m \\ d_t^c \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} d_{t-1}^m \\ d_{t-1}^c \end{pmatrix} + \begin{pmatrix} \varphi_t \\ \eta_t \end{pmatrix}$$

In compact form as:

$$\tilde{P}_t = HZ_t \qquad \tilde{P}_t = P_t - P_{t-1}$$

$$Z_t = AZ_{t-1} + \delta_t \qquad \delta_t \sim \mathrm{N}\left(0, Q\right)$$

where $\tilde{P}_t$ is the observed asset price (first-difference),

$$Z_t = \begin{pmatrix} d_t^m \\ d_t^c \end{pmatrix}$$

is the state vector,

$$H = \begin{pmatrix} \omega & 1-\omega \end{pmatrix}$$

is the measurement matrix,

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

is the transition matrix and δ_t is the vector containing the state disturbance, with variances collected in the matrix Q.

With a state-space model, we can reveal the nature and the cause of the dynamic movement of observed variables. Indeed, it is possible to explain an observed variable's behavior by examining the unobserved components' internal dynamics. In our case, we can analyze the effect of the heuristic beliefs (unobserved components variables) on asset price (observed variable).

The system's dynamics is obtained from the transition equation, which describes the evolution of the vector of unknown latent variables. Eigenvalues analysis is performed to study the oscillation conditions associated with the unobserved beliefs. Solving determinant of the transition matrix, we obtain the characteristic equation:

$$\begin{vmatrix} a_{11}-\lambda & a_{12} \\ a_{21} & a_{22}-\lambda \end{vmatrix} = 0$$

so that

$$\begin{vmatrix} a_{11}-\lambda & a_{12} \\ a_{21} & a_{22}-\lambda \end{vmatrix} = (a_{11}-\lambda)(a_{22}-\lambda) - a_{12}a_{21} = 0$$

from which

$$a_{11}a_{22} - \lambda a_{11} - \lambda a_{22} + \lambda^2 - a_{12}a_{21} = 0$$

i.e:

$$\lambda^2 - \lambda Tr(A) + Det(A) = 0$$

so to obtain

$$\lambda_{1,2} = \frac{Tr(A) \pm \sqrt{Tr(A)^2 - 4Det(A)}}{2}$$

Oscillations can be expressed in terms of the negative discriminant:

$$\Delta = Tr(A)^2 - 4Det(A) < 0$$

i.e.:

$$(a_{11} - a_{22})^2 + 4a_{12}a_{21} < 0 \tag{13}$$

where the necessary condition is

$$a_{12}a_{21} < 0$$

If Eq. (13) holds:

$$\lambda_{1,2} = \frac{Tr(A)}{2} \pm i\frac{\sqrt{-\Delta}}{2} = a \pm ib$$

or in the equivalent trigonometric form [16]:

$$\lambda_{1,2} = \rho(\cos\theta \pm i\sin\theta)$$

.

To obtain oscillations of constant amplitude, we require:

$$\rho = \left(a^2 + b^2\right)^{\frac{1}{2}} = 1$$

where ρ is the modulus of the complex number.

With respect to the parameters of interest, we obtain:

$$\sqrt{\frac{(a_{11}+a_{22})^2}{4} + \frac{-(a_{11}+a_{22})^2 + 4(a_{11}a_{22} - a_{12}a_{21})}{4}} = 1$$

i.e.:

$$\sqrt{(a_{11}a_{22} - a_{12}a_{21})} = 1$$

If the condition in Eq. (13) holds, with $\sqrt{(a_{11}a_{22} - a_{12}a_{21})} < 1$ ($\rho < 1$), we observe damped oscillations. With $\sqrt{(a_{11}a_{22} - a_{12}a_{21})} > 1$ ($\rho > 1$), we observe explosive oscillations.

Finally, using the complex eigenvalues in the trigonometric form, the implied length of the cycles is given by

$$\frac{2\pi}{\theta} = \frac{2\pi}{\arccos\left({}^{a}\!/_{\rho}\right)} \tag{14}$$

3 Data and estimation method

Data on equity prices of the US, France, Germany, and the UK are used in the estimation procedure. The source of time series is the database of OECD statistics where these indices are determined by the stock exchange using the closing adjusted values. All the series are deflated with the country-specific CPI-index in the FRED database, and logarithms are taken. Our final sample comprises the 1960–2020 period.

Once the model is in a state-space form, we apply the iterative prediction error decomposition approach for identifying the maximum likelihood estimates of parameters; the one-step prediction and updating equations are calculated in a state-space form using the iterative Kalman filtering. The Kalman filter is a recursive dynamic procedure for calculating the optimal estimator of the unobserved state vector. The goal is to minimize the mean square prediction error of the unobserved state vector conditional of the observation of $\tilde{P}_t$ (See Appendix A).

This procedure allows us to directly estimate the cyclical parameters $[a_{11}, a_{12}, a_{21}, a_{22}]$ and the percentage of the two groups of agents in the market $[\omega, 1-\omega]$. After obtaining the estimates, we check to see if the cyclical condition is satisfied $[(a_{11} - a_{22})^2 + 4a_{12}a_{21} < 0]$. Finally, we recover the positive and negative signs of chartists' coefficients from the necessary condition parameters $[a_{12}, a_{21}]$. All the analyses are checked with diagnostic tests on the standardized prediction errors, and using complex values in a trigonometric form, we extract the period of cycles.

Regarding this last point, in line with parametric unobserved component models, and as opposed to non-parametric filters, no prior assumptions on the cycle's length have been imposed [8]. Moreover, unlike the parametric filtering procedure, where the cycle is formalized as an autoregressive trigonometric process of order two with the roots lying in the complex plane, we do not impose any conditions for the cyclical component [19, 15, 5].[4] In other words, we do not impose any conditions to obtain complex eigenvalues from the polynomial characteristic. If we obtain cycles, these are the results of the unobserved strategies adopted by the two groups of agents formalized as unobserved component variables.

[4]In the standard approach, the cyclical dynamics is parameterized in terms of cycle length and persistence [5]. Usually, the period across countries ranges from 5.8 to 6.6 years.

4 Results

This section sets out the results of our examination.

Table 1 shows the estimation results for the countries considered. The first left column exhibits the parameters of interest with the cyclical condition. The central columns refer to our four countries: France, Germany, the UK and the USA. As we can see, for all four countries, the cyclical condition is respected $[(a_{11} - a_{22})^2 + 4a_{12}a_{21} < 0]$. For Germany and the USA, the parameters a_{11}, a_{12}, a_{21} and a_{22} are statistically significant at 1% level. For France, a_{22} is significant at 10%, while for the UK, it is not statistically significant; a_{11}, a_{12}, a_{21} remain statistically significant at 1% level.

Concerning the percentage of agents in the market, we observe significant evidence for the existence of the two types of agents. We notice that the percentage of contrarians is higher than the percentage of trend followers in the European countries. In France, contrarians (trend followers) are estimated to be 68% (32%), followed by 70% (30%) in Germany, and 63% (37%) in the UK. Differently, in the USA, trend followers predominate over contrarians. In this case, trend followers are estimated to be 65% while contrarians are 35%. From these results, we notice a difference between European countries and the USA. In the European countries, the majority do not share the speculative position of trend followers, while the opposite is true for the USA. However, in the European countries, this effect is compensated by an higher price overshooting. For contrarians, we find ϕ equal to -1.7 for Germany, $\phi = -1.5$ for France, and finally $\phi = -1.4$ for the USA and the UK. In terms of β values, the highest price overshooting is in the USA, with β equal to 2.2. Next is the UK ($\beta = 2.1$), followed by Germany ($\beta = 1.9$), and France ($\beta = 1.6$).

Overall, the estimation of the univariate model leads to cyclical fluctuations. More specifically, we find empirical evidence of endogenous financial fluctuations due to the two behavioral rules defined in our model. The highest price overshooting is in the two most advanced financial market-oriented economies, the USA and the UK, where we observe a larger coefficient magnitude. Moreover, the USA and the UK also have higher percentages of trend followers than France and Germany.

The obtained results are controlled with residuals diagnostic checks. We check residuals for autocorrelation, conditional heteroscedasticity, and normality. Specifically, we conduct the Ljung-Box Q-test, Engle's ARCH test, and Jarque–Bera on the vector prediction errors. The first test checks the null hypothesis of jointly zero autocorrelations against the alternative of at least one nonzero autocorrelation. The second one tests the null hypothesis of no ARCH effects against the alternative ARCH model. Finally, the Jarque–Bera test checks for the null hypothesis that the error series comes from a normal distribution.

The results of residual diagnostics are depicted in Table 2. For all the countries considered, we do not reject the hypothesis of no serial correlation; the sample autocorrelations in Figures 1 and 2 confirm this up to the lag 20. As for autocorrelation, there is no heteroscedasticity problem; the statistical value is lower than the critical value. Finally, we do not have a problem with normality for Germany and the UK, while the normality test is not supported for France and the USA.

Table 1: Estimation via Kalman filter for equity prices

	France	Germany	UK	USA
a_{11}	1.00142***	1.17172***	1.10691***	0.73490***
	(0.11600)	(0.15805)	(0.07800)	(0.11616)
a_{21}	-0.47666***	-0.52135***	-0.53106***	-0.95336***
	(0.03851)	(0.02909)	(0.04437)	(0.20550)
a_{12}	1.08472***	1.33226***	1.34825***	0.78403***
	(0.08237)	(0.37942)	(0.44079)	(0.03075)
a_{22}	-0.12794*	-0.33133***	-0.28004	-0.73421***
	(0.07128)	(0.08066)	(0.21346)	(0.01526)
	Cyclical Conditions			
$[(a_{11} - a_{22})^2 + 4a_{12}a_{21} < 0]$	*yes*	*yes*	*yes*	*yes*
ω	0.32***	0.30***	0.37***	0.65***
	(0.01396)	(0.01173)	(0.01187)	(0.0013)
$1 - \omega$	0.68***	0.70***	0.63***	0.35***
	(0.01396)	(0.01173)	(0.01187)	(0.0013)
β	1.6	1.9	2.1	2.2
ϕ	-1.5	-1.7	-1.4	-1.4
σ_ε	0.21729***	0.21838***	0.16614***	0.05787***
	(0.04342)	(0.04116)	(0.02190)	(0.00811)
σ_η	0.10758***	0.09422***	0.09333***	0.12588***
	(0.01006)	(0.03114)	(0.01585)	(0.01545)
Log-likelihood	50.3119	56.0239	61.7598	83.2709
Sample	1960-2020	1960-2020	1960-2017	1960-2020

Notes: Standard errors in parentheses.

*, **, *** denotes statistical significance at the 10%, 5%, and 1% levels respectively.

Table 2: Diagnostic checks for equity prices

	France	Germany	UK	USA
		Autocorrelation Test		
Pvalue	0.4407	0.3196	0.9978	0.5290
CValue	37.56	37.56	37.56	37.56
Stat	20.27	22.39	6.56	18.88
		Heteroscedasticity Test		
Pvalue	0.7640	0.6906	0.5587	0.6909
CValue	37.56	37.56	37.56	37.56
Stat	15.21	16.41	18.43	16.41
		Normality Test		
Pvalue	0.0054	0.0841	0.4255	0.0042
CValue	12.52	12.52	12.52	12.52
Stat	16.72	3.70	1.24	18.74

Notes: *Cv* and *Stat* are respectively the critical value and the test statistics.

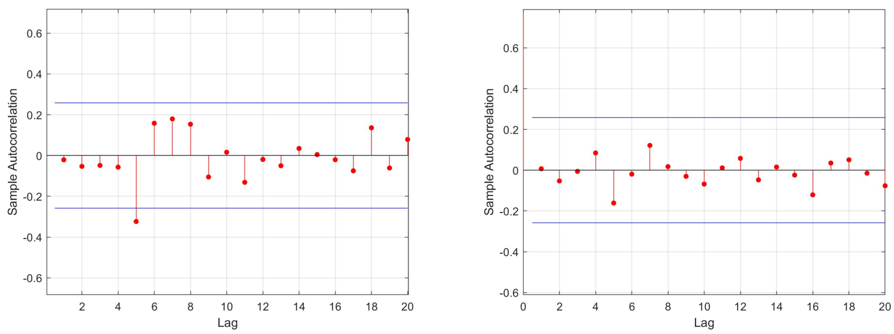

Figure 1: Sample Autocorrelation for USA (left) and UK (right).

The qualitative difference in the dynamics of the unobserved state components can be observed in Figures 3, 4, 5, and 6. The dynamics of the estimates is obtained from the one-side filtering procedure. In black, we see the dynamics of the excess demands for trend followers. In red, we have the excess demand of contrarians, while in cyan, we have the dynamics of asset prices in the first difference. The figures show several interesting dynamics. As we can see, the excess demands of the two groups of agents are of opposite signs. This depends on the agents' different expectations formation, which are reflected in the excess demands and, consequently, in determining the asset price dynamics. In particular, we notice an increasing positive excess demand of trend followers before the dot.com crisis (2000) and the global financial crisis (2007–2008). These years are characterized by an increasing wave of optimism in the financial system market; the sign of the dynamics changes after the bubbles explosions. Moreover, the trend follower effect (black color) is much more pronounced for the crisis of

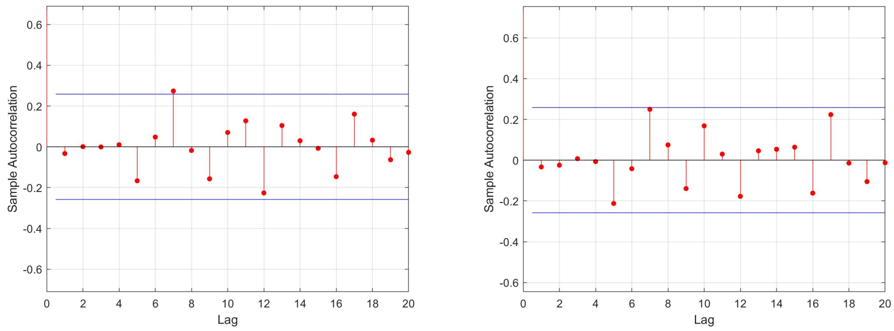

Figure 2: Sample Autocorrelation for France (left) and Germany (right).

2000 (a stock market crisis) compared to the global financial crisis (a bank sector crisis).

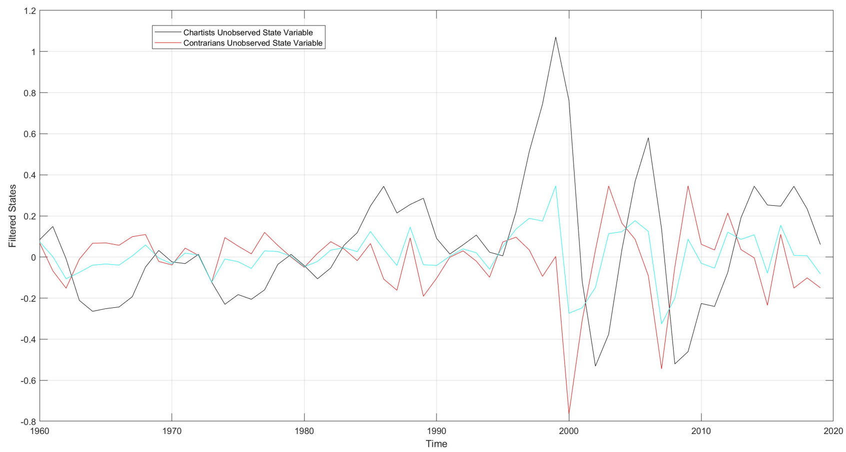

Figure 3: Filtered Unobserved State Dynamics for France

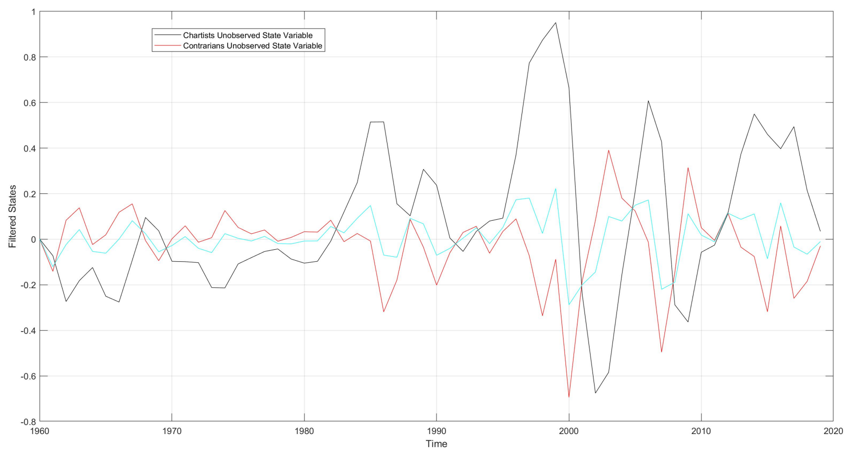

Figure 4: Filtered Unobserved State Dynamics for Germany

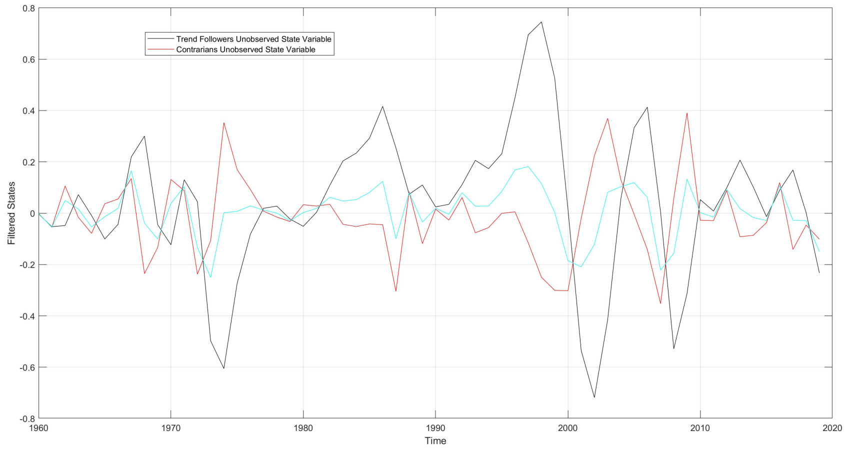

Figure 5: Filtered Unobserved State Dynamics for UK

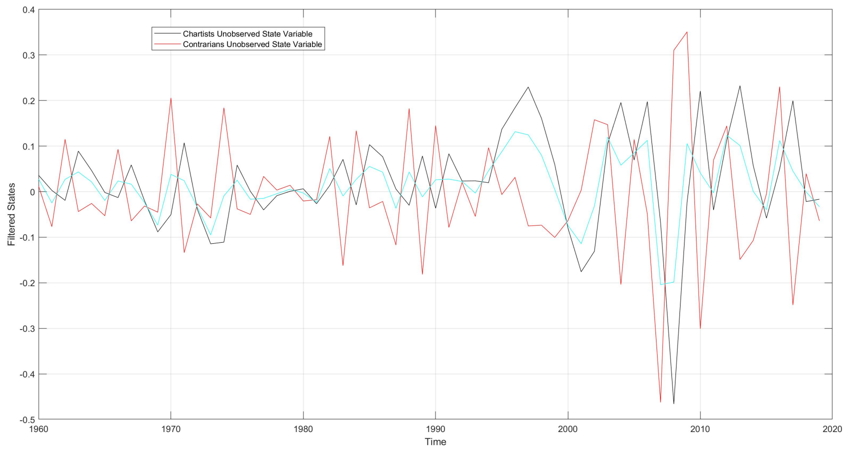

Figure 6: Filtered Unobserved State Dynamics for USA

Finally, using Eq. (14), we extract the length of the endogenous financial cycles. As confirmed by previous research, equity price frequencies are higher than the frequencies of other financial variables [2]. Looking at the cycle length implied by the eigenvalues of the estimated coefficient matrix, the financial cycles have estimated lengths of four years for the USA, seven years for the UK, and eight years for France. These results are in line with the literature on equity price financial cycles [15]. By contrast, in the case of Germany, the data does not appear to give rise to short cycles. The estimate appears with a more extended period of nine years.

From the obtained results, we can grasp two final considerations. Firstly, when the cyclical phenomena are the consequence of behavioral strategies adopted by the agents, the implied cycle length is short. Second, even with some differences, results are quite similar for the countries considered. This is true not only for the implied lenght cycles but also for the reaction coefficients. These results confirm the presence of a sentiment spillover in international stock markets to generate a global financial cycle. At the same time, the results align with previous findings that show how equity price cycles are both more synchronous and more similar across countries than other series [33].

Table 3: Period of endogenous cycles

	France	Germany	UK	USA
Eigenvalues	$0.4367 \pm 0.4452i$	$0.4202 \pm 0.3603i$	$0.4134 \pm 0.4849i$	$0.0003 \pm 0.4560i$
Modulus	0.62	0.55	0.63	0.45
Period	8	9	7	4

Notes: Periods of cycles in years.

5 Conclusions

Recent empirical works have confirmed the importance of behavioral heuristics in asset pricing. We approach the view of financial fluctuations as the possible result of behavioral forces that perpetuate endogenous cycles phenomena. The contribution of this paper is twofold. First, we put the heterogeneous agents setting in a state-space form. This methodology enables us to identify the system's mathematical conditions to obtain fluctuations phenomena and see if the cyclical conditions are empirically respected. Second, we extract the implied length cycles within a simple analytical framework of heterogeneous agents. For the USA, France, and the UK, we show that speculative sentiment can drive financial cycles to generate short-term cycles, confirming that sentiment dynamics influence stock markets.

One main strand of this thesis can be further explored. A one-market HA model can be extended to multiple markets. In particular, the analysis can be extended to consider the interactions between real and financial sectors. In this case, a multivariate model could be constructed to study those interactions. More specifically, this type of extension could allow us to understand the interaction between the speculative positions adopted in the two markets and to study the nature of possible correlations. We leave this extension to future research.

In conclusion, for policymakers to better understand and potentially take measures against financial instability, the focus should be on the interaction between market participants moving away from model-consistent "rational" expectations. Deliberately, this paper does not explore these important policy-related issues in depth. Rather, it is designed to convey that the link between the asset price dynamics and the policy response is not neutral. At the same time, however, this requires a change of theoretical view. It will be necessary to pass from an equilibrium analysis to an analysis of disequilibrium in which heuristic decisions are the leading cause of the complex nature of the economy.

Appendix A

The optimal forecasting rule has the form

$$Z_{t\,|\,t} = Z_{t\,|\,t-1} + K_t\left(\tilde{P}_t - \tilde{P}_{t\,|\,t-1}\right)$$

where K_t is a weight that changes as new information becomes available, $Z_{t\,|\,t}$ denotes the forecast of state variable once $\tilde{P}_t$ is realized while $Z_{t\,|\,t-1}$ and $\tilde{P}_{t\,|\,t-1}$ denote respectively the forecast of variables Z_t and $\tilde{P}_t$ before $\tilde{P}_t$ is realized.

Now we can select the optimal value of K_t to minimize the mean square prediction error at time t

$$\min_{k_t} E_t\big(Z_t - Z_{t\,|\,t}\big)^2 = \min_{k_t} E_t\Big[Z_t - \Big(Z_{t\,|\,t-1} + K_t\left(\tilde{P}_t - \tilde{P}_{t\,|\,t-1}\right)\Big)\Big]^2$$

We obtain:

$$\min_{k_t} E_t\big[Z_t - \big(Z_{t\,|\,t-1} + K_t\left(HZ_t - HZ_{t\,|\,t-1}\right)\big)\big]^2$$

$$\min_{k_t} E_t\big[(I - HK_t)\left(Z_t - Z_{t\,|\,t-1}\right)\big]^2$$

$$\min_{k_t} (I - HK_t)^2 E_t\big(Z_t - Z_{t\,|\,t-1}\big)^2$$

Optimizing with respect to K_t we get

$$-2H\left(I - HK_t\right)E_t\big(Z_t - Z_{t\,|\,t-1}\big)^2 = 0$$

Indicating with $\Gamma_{t\,|\,t-1} = E_t\big(Z_t - Z_{t\,|\,t-1}\big)^2$, we obtain

$$-2H\left(I - HK_t\right)\Gamma_{t\,|\,t-1} = 0$$

Solving for K_t we obtain

$$K_t = \frac{H\ \Gamma_{t\,|\,t-1}}{H\ \Gamma_{t\,|\,t-1}H'}$$

Regrouping the equations, we obtain that

$$Z_{t\,|\,t-1} = AZ_{t-1\,|\,t-1} \tag{A.1}$$

$$\Gamma_{t\,|\,t-1} = A\,\Gamma_{t-1\,|\,t-1}A' + Q \tag{A.2}$$

$$\tilde{P}_{t\,|\,t-1} = H\tilde{P}_{t-1\,|\,t-1}$$

Eqs. (A.1) and (A.2) are the so-called prediction equations in the Kalman filtering. The other equations we need are the three updating equations which are

$$K_t = \Gamma_{t\,|\,t-1}H'(\psi_t)^{-1} \tag{A.3}$$

with

$$\psi_t = H\Gamma_{t\,|\,t-1}H'$$

$$Z_{t\,|\,t} = Z_{t\,|\,t-1} + K_t\left(\tilde{P}_t - \tilde{P}_{t\,|\,t-1}\right) \tag{A.4}$$

$$\Gamma_{t\,|\,t} = (I - K_tH)\,\Gamma_{t\,|\,t-1} \tag{A.5}$$

In this case, the inference about Z_t is updated using the observed value of $\tilde{P}_t$.

We start with a specification information set with initial conditions $Z_{0\,|\,0}$ and $\Gamma_{0\,|\,0}$. Then we use the prediction equations (A.1) and (A.2) to obtain $Z_{1\,|\,0}$ and $\Gamma_{1\,|\,0}$. Once we observe $\tilde{P}_1$ we use the updating equations (A.3), (A.4), and (A.5) to obtain $Z_{1\,|\,1}$, $\Gamma_{1\,|\,1}$ and $\tilde{P}_{1\,|\,1}$. We next use this information to form $Z_{2\,|\,1}$ and $\Gamma_{2\,|\,1}$, then forecasts are updated and we continue to repeat this process until the end of the dataset.

Given the vector prediction errors $\mu_t = \tilde{P}_t - \tilde{P}_{t\,|\,t-1}$ and the variance-covariance matrix ψ_t, we can form the log-likelihood to be maximized and to estimate our parameters.

$$\log l = -\tfrac{T}{2}\ln(2\pi) - \tfrac{1}{2}\sum_{t=1}^{T}\ln\left(\left|\psi_{t\,|\,t-1}\right|\right) - \tfrac{1}{2}\sum_{t=1}^{T}\mu_t{}'\left(\psi_{t\,|\,t-1}\right)^{-1}\mu_t$$

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