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Intellectual Property and Taxation in Digital Platforms

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Abstract

I study the impact of competition and taxation on the openness and the intellectual property policies of two-sided digital platforms. I model a market in which two platforms compete for users and developers. First, I find that higher competition shortens the period of exclusivity granted to developers but does not influence the degree of openness of a platform. However, the higher the degree of differentiation in the developers market, the less open the platforms are. Second, I analyze two types of taxes, ad valorem and unit taxes. Ad-valorem taxes have no effect on the length of the exclusivity period. However, they increase the degree of openness of the platform when levied on users. Unit taxes instead limit the degree of openness and increase the period of exclusivity when levied on the developers market. Lastly, I find that multi-homing reduces the exclusivity period, but does not change the qualitative effects of taxation. My findings suggest that the new digital tax proposed by the European Commission, that should come into force in 2020, may reduce openness and innovation levels in the European Union.

Keywords— Two-sided markets, Digital Platforms, Taxation, Intellectual Property, Openness
JEL-Codes— H22, L13, L51, L86, O34

1 Introduction

Discussions on the taxation of the digital economy have intensified at the European level since September 21, 2017. That day, the European Commission issued a communication which presented the challenges in taxing digital businesses. On March 21, 2018, the European Commission took a step forward by presenting a Digital Service Tax (DST) that will come into force in 2020.

Despite considerable research on the taxation of digital platforms, to the best of my knowledge, no formal analysis investigates how taxes may affect the the intellectual property (IP) rights of platforms and third-party developers. It is well known that taxation distorts the investment incentives, and therefore that may hamper the innovations carried out by platforms. However, up to the date, we do not know if taxation is hampering the innovations generated by third-party developers, or if it is modifying the openness level of digital platforms.

I study the impact of competition and taxation (an added-value tax and a unit tax) on the degree of openness of platforms and on the period that developers can keep the intellectual property of their innovations. I find that competition decreases the exclusivity period awarded to developers to exploit their innovations, but it has no effect on the openness. On the other hand, I find that higher market differentiation implies larger exclusivity periods but platforms less open.

I also find that ad-valorem taxes levied on the royalties paid by developers have no effect on the length of the exclusivity period, but the higher the ad-valorem taxes, the lower the openness. The opposite is true if such tax is levied on users. In the case of unit taxes levied on revenues from developers, they increase the period of exclusivity and reduce the openness. However, if such taxes are levied on revenues from users, platforms become more open.

I contrast these results in an environment with developers who demand both platforms at the same time (multihoming), and I find that multihoming reduces the exclusivity period of developers, but the intuitions about taxation are unchanged. Lastly, I compare the previous equilibria with the case in which platforms have limited knowledge about their demands, and I prove that the intuitions with full information are still valid.

I conclude by highlighting that taxation of digital platforms modifies the optimal IP policies of platforms. It matters who pay. If we tax developers or advertisers activities, platforms will be less open. Therefore, if the DST comes into force, I expect a lower level of openness and innovation in the European ecosystem.

2 Taxation of Multi-sided Platforms

Taxation of two-sided digital markets has attracted the attention of scholars after the seminal contribution of Kind et al. (2008). A decade after, there is a renewed attention in this field as a consequence of the growth of

these platforms and their capacity to influence the economy[1]. At this moment, the literature agrees on that tax authorities should reform and adapt their instruments to take into account the new conditions created by the emergence of the digital economy, see Bacache et al. (2015).
The effect on prices is the most studied topic. In this field, there are enough pieces of evidence about the consequences of taxation on two-sided digital platforms. For example, Kind et al. (2008) and Kind et al. (2010) find that a higher value-added tax on one side may make profitable for the platform to shift revenue from that side to the other one (from the heavily taxed to the untaxed side). These authors also find that the dominance of ad-valorem taxes that is common in one-sided markets does not hold in two-sided markets. In fact, unit taxes may yield higher welfare than ad-valorem taxes, see Kind et al. (2009)[2].
Recently, Belleflamme and Toulemonde (2018) have found that transaction taxes hurt agents on both sides but benefit platforms. On the other hand, they also confirm the result found by Kind et al. (2008) that ad valorem taxes may benefit the agents that are taxed, but it may hurt agents on the other side of the market[3].
Additionally, in a recent literature review, Bacache et al. (2015) point out that taxation distorts the investment decisions of platforms. Hence, it may hamper innovation. In that sense, it is imperative to keep a careful watch on the evolution of internet platforms.
Although the role of taxes is not new, there are some questions that remain unanswered[4]. The impact of those taxes on the IP policies and the openness of platforms has not been addressed, and the results presented up to date point out that the innate characteristics of digital products and digital platforms modify the effects and the optimal design of taxes in potentially unexpected ways, see Kind and Koethenbuerger (2018).
Nonetheless, at the same time that the European Union is paying attention to the taxation of these platforms, it is also paying attention to foster the ecosystems based on the open source paradigm. However, there has been no initiative interested in addressing these two areas from an economic perspective. My contribution is precisely to spark the discussion about the role of taxation in the openness and the design of IP policies in two-sided digital platforms.
In this sense, to the best of my knowledge, only Parker and Van Alstyne (2017) address the innovation decisions of platforms. They propose a monopoly platform that chooses how much intellectual property to give away and how long developers can keep the innovations generated upon the platform. From this starting point, I extend their model to address the impact of competition and taxation on a duopoly market.

3 The Fundamentals of Parker and Van Alstyne Model

My model is based on Parker and Van Alstyne (2017)[5]. This model encompasses a developer and a consumer that get in touch on a digital platform during two stages. The developer produces apps using the platform resources, such as application programming interfaces (APIs) or system developer toolkits (SDKs). The consumer uses both, the platform and the developer's apps.
The intuition of the model is the following. By giving away some IP rights, the platform offers some technologies that the developer can use to innovate upon. In return, the developer shares with the platform part of its revenues by paying royalties. After a certain amount of time, the platform absorbs the developer's IP rights and the pool of technologies available to innovate upon grows (cumulative innovation).
The model encompasses two stages. In the first stage, the platform sets the proportion of the platform that is open to the developer (σ, the degree of openness)[6] and the time awarded to the developer to exploit its innovations (t). The developer sells its apps to the consumer, and it pays royalties to the platform. At the same time, the consumer pays a fee for using the app, and she pays for the non-open part of the platform. At the end of this stage, the platform absorbs the innovations that are incorporated into the pool of technologies that will be awarded to the developer in the next stage to innovate upon. The period that the developer has exclusive rights over its innovations defines the length of this stage. If $t = 0$, innovations are immediately absorbed, if $t = \infty$ no innovation will be absorbed. At the second stage, the developer innovates on top of the pool of technologies that the platform has, and again, the developer sells apps to the consumer and pays royalties to the platform.
The consumer has a uniform value v for developers' apps and a uniform value V for the platform. Nonetheless, the consumer can wait until the second stage to acquire developers' apps for free because of the acquisition by the platform. Therefore, the consumer is not willing to pay more than the difference between the value today (v) and the discount value of waiting to the next stage (δv). The maximum price she is willing to pay is $p = (1 - \delta)v$. On the other hand, the platform can sell its own technology to the consumer (V), but if it sets a

[1] See Belleflamme and Toulemonde (2018) for a short review on this topic.
[2] In a recent work, they also find that taxes may affect the political views of newspapers, see Kind et al. (2013).
[3] Other interesting works in this area are Bourreau et al. (2018), Kind et al. (2010) and Tremblay (2018).
[4] For example, Bacache et al. (2015) state that the current academic economic literature on e-commerce and its tax system remains relatively scarce.
[5] For simplicity's sake, I also keep the same notation.
[6] The openness could be higher than 1, that implies that the platform is subsidizing the developer.

positive σ, its sales fall to $(1-\sigma V)$ because part of the technology is given for free to the developer[7].
The developer generates innovations (apps) following a Cobb-Douglas production function ($y = k(\sigma V)^{\alpha}$). Where σV represents the open resources of the platform, k is the re-usability parameter, and α represents the diminishing returns of the technology. The output of the first stage is $y_1 = k(\sigma V)^{\alpha}$ and the output of the second stage is $y_2 = k(y_1)^{\alpha}$. Note that innovations are recursive, they are generated on top of previous innovations.
Lastly, the platform imposes a royalty on the developer's revenues. For simplicity's sake, Parker and Van Alstyne assumes a the Nash bargaining solution, giving each party 50%[8]. Therefore, the profits of the platform and the developer are respectively

$$\Pi = V - \sigma V + \frac{1}{2}(py_1 + \delta py_2) \tag{3.1}$$

$$\pi = \frac{1}{2}(py_1 + \delta py_2) \tag{3.2}$$

The platform faces two trade-offs. One the one hand, it should decide how much intellectual property it gives away. By giving away intellectual property, it increases the pool of technologies upon the developer can innovate, and therefore, the more valuable the innovations at the second stage, and the larger the royalties. However, not giving away intellectual property allows the platform to monetize its technologies.
On the other hand, it must set how long is the period to exploit developers' innovations before they are integrated into the pool of technologies of the platform. The longer the developer keeps its rights, the higher the revenues that the platform and the developer earn in the first stage. In contrast, the sooner the technologies are absorbed, the sooner the developer can innovate upon the new set of technologies, the larger the added-value generated in the second stage.
This behavior is common in digital markets. Examples of this trade-off can be found on many digital platforms, such as Facebook, Google, Apple, or eBay[9]. All of them offer APIs to developers that can use to innovate, and from time to time, they acquire some technologies by merger or acquisition. See, for example, the acquisition of Whatsapp by Facebook, or the acquisition of Paypal by eBay.
Using this framework, I extend the Parker and Van Alstyne's model to address the impact of competition and taxation in a duopolistic market with heterogeneous developers and homogeneous consumers.

4 The role of competition and strategic interactions: the duopoly

In the original model, Parker and Van Alstyne consider a market with one platform, one developer, and one user. Nonetheless, I relax such assumptions, and I consider there are two platforms ($j = 1, 2$), a set of homogeneous users[10], and a set of heterogeneous developers à la Hotelling. Additionally, to avoid the possibility of developers changing from one platform to another at the second stage, I assume that switching costs are extremely high[11].
In comparison with the original framework, my framework modifies the production function at the second stage,

$$y_{2,j} = k(\sum y_{1,j}) = k^{1+\alpha} m_{e,j}^{\alpha} (\sigma_j V_j)^{\alpha^2}$$

Where $m_{e,j}$ is the expected number of developers that contributed to the platform in the first stage. The developers' profits are also modified in our framework. Developers are heterogeneous à la Hotelling, and therefore, they face transportation costs, $t|x_i - l_j|$, where $(l_j, x_i) \in [0,1]$, t is the transportation cost, x_i represents the developers' position in the Hotelling segment, and l_j represents the platforms' position in the same segment.
The intuition is the following. When a developer evaluates the possibility of joining one of the platforms, they address the profits they would earn in the two periods, and the transportation costs derived from being on one of the platforms. Therefore, the developers' utility function is

$$U_{i,j}^{d} = \frac{p_j^e k(\sigma_j V_j)^{\alpha} + \delta_j p_j^e m_{e,1}^{\alpha} k^{\alpha+1} (\sigma_j V_j)^{\alpha^2}}{2} - t|x_i - l_j| \tag{4.1}$$

For simplicity's sake, I denote $\pi_{1,j} = p_j^e k(\sigma_j V_j)^{\alpha}$ and $\pi_{2,j} = p_j^e m_{e,1}^{\alpha} k^{\alpha+1} (\sigma_j V_j)^{\alpha^2}$. I make another modification with respect to Parker and Van Alstyne's model, I assume the developers have some expectations about the

[7]Note that $\delta = e^{-rt}$.

[8]I keep the same assumption.

[9]Nowadays, Google or Apple themselves generate fewer innovations than the Android or iOS ecosystems. It has become more relevant the investment carried out by independent developers on the platform than the investment carried out by the owner of the platform. See for example Uber, Tinder, Airbnb or Whatsapp.

[10]The presence of heterogeneous users does not change our conclusions because developers are the ones who set the prices for their apps, not the platforms. This is a realistic approach since Google Play or the App store do not set the price for third-party apps.

[11]I assume the market is complete and all developers want to stay in the market.

price of their production (apps) on the platforms (p^e_j)[12]. However, I assume no specific process of expectation formation neither here nor in the expected number of developers on platforms[13]. Solving $U^d_{i,j} \geq U^d_{i,-j}$, I obtain the demand expressions for both platforms[14].

$$M^d_j = \frac{1}{2} + \frac{\pi_{1,j} - \pi_{1,-j} + \delta_j \pi_{2,j} - \delta_{-j}\pi_{2,-j}}{4t} \tag{4.2}$$

I have assumed that, when developers are making their decisions, they do not know how platforms will behave, and therefore they have expectations about the number of developers on the platform, and the prices they will set. In the same way, platforms do not know how many developers will join, but they know how their policies will influence developers pricing decisions. Platforms anticipate that developers will set $p_j = v(1-\delta_j)$. The intuition is the following. The people behind platforms are also developers. Therefore, it makes sense to think that the developers working in the platform know how other developers will behave under specific market policies. What they cannot anticipate is the adoption of the platform[15]. In this scenario, the platforms choose the degree of openness and the duration of the exclusivity contract simultaneously, formally

$$\max_{\sigma_j,\delta_j} \Pi_j = V_j n^e_j (1-\sigma_j) + \frac{1}{2} M^d_j p_j \tag{4.3}$$

Where n^e_j is the expected number of users that will adopt the platform. Note that the first term in Eq. 4.3 represents the revenues from users, and the second term represents the revenues from developers. Platforms profits Π_j are well behaved and there exists a unique pair $\langle \sigma^*, \delta^* \rangle$ that maximizes π_{plat}. Nonetheless, I find two solutions, a corner solution, and an interior solution. Which one will be the equilibrium depends on the size of the difference of the outputs of the platforms at the first stage adjusted by the transportation costs with respect to the total production of the platforms at the second stage.

$$\delta^*_j = \begin{cases} \frac{1}{2}\left[1 - \frac{\pi^d_{1,j} - \pi^d_{1,-j} + 2t - \delta_{-j}\pi^d_{-j,2}}{\pi^d_{j,2}}\right] & \text{if } 2\pi^d_{j,2} + \pi^d_{-j,2} > \Delta^1 + 6t \\ 0 & \text{otherwise} \end{cases} \tag{4.4}$$

Where Δ^1 represents $(\pi^d_{1,j} - \pi^d_{1,-j})$. It is interesting to point out that my result shows a clear link with the Parker and Van Alstyne's model that can be observed in the interior solution.

$$\frac{1}{2}\left[\underbrace{1 - \frac{\pi^d_{1,j}}{\pi^d_{j,2}}}_{\text{PV(2017)}} \underbrace{\frac{-\pi^d_{1,-j} + 2t - \delta_{-j}\pi^d_{-j,2}}{\pi^d_{j,2}}}_{\text{Duopoly effects}}\right]$$

This expression points out that, when choosing how long should be the period of exclusivity awarded to developers, platforms take into account what other platforms do. In this case, they influence each other, and that influence is positive. Competition in δ leads to shorter periods of exclusivity (bigger δ). Solving the system of equations $\delta_j(\delta_{-j}), \delta_{-j}(\delta_j)$,

$$\delta^*_j = \begin{cases} \frac{1}{3}\left[\frac{2\pi^d_{j,2} + \pi^d_{-j,2} - \Delta^1 - 6t}{\pi^d_{j,2}}\right] & \text{if } 2\pi^d_{j,2} + \pi^d_{-j,2} > \Delta^1 + 6t \\ 0 & \text{otherwise} \end{cases} \tag{4.5}$$

On the other hand, the optimal solution of σ depends on the equilibrium solution of δ. Therefore, two cases are possible.

$$\sigma^*_j = \begin{cases} \frac{\alpha(\pi^d_{1,j} + \delta_j \alpha \pi^d_{2,j})}{4t} \frac{v(1-\delta^*_j)}{2V_j n^e_{j,1}} & \text{if } \delta^*_j > 0 \\ \frac{(v\alpha p^e_j k/8t)^{1/(1-\alpha)}}{V_j n^e_{j,1}} & \text{if } \delta^*_j = 0 \end{cases} \tag{4.6}$$

In this case, it is interesting to point out that platforms do not take into account what their competitors do (except throughout the δ). Nonetheless, this is a consequence of one implicit assumption. I assume $\frac{\partial \sigma_j}{\partial \sigma_{-j}} = 0$.

[12] Note that σ can be higher than one, but it cannot be lower than 0, which implies the platform subsidize users. In this framework, a negative σ would imply that developers produce negative quantities, which is not possible. On the other hand, I assume developers always set a non-negative price for their apps (e.g. pay-to-play games).

[13] I want to stress the role these variables play in the platform decision process. If I assume one specific expectation formation process, it leads us to complex analytical expressions that do not help us in having a clear picture. Additionally, the comparison with the Parker and Van Alstyne's model becomes blurry.

[14] Symmetrically, M^d_{-j}.

[15] This is not a strong assumption. Developers sell apps in a one-sided market with homogeneous users.

It is unknown to what extent this is a realistic depiction of the market. Intuitively, I have reasons to think that openness decisions are independent of each other, and they depend more on the culture of the company that supports the platforms than on the market itself. If we pay attention to Android or iOS, we observe that their policies regarding the use of those systems differ significantly. For example, iOS has kept the same integrated structure regarding their openness since the launch of the first iPhone, despite numerous changes in the competition in the market. Although their policies have changed during this period of time, it seems that such changes were a consequence of other market features[16]. Nonetheless, my σ interior solution also show a clear link with the Parker and Van Alstyne results.

$$\begin{array}{ll} \overbrace{\frac{\alpha(\pi^d_{1,j}+\delta_j\alpha\pi^d_{2,j})}{V_j}}^{\text{PV(2017)}} \; \overbrace{\frac{v(1-\delta_j^*)}{8tn^e_{j,1}}}^{\text{Duopoly effects}} & \text{if } \delta_j^*>0 \\ \underbrace{\frac{(v\alpha k/2)^{1/(1-\alpha)}}{V_j}}_{\text{PV(2017)}} \; \underbrace{\frac{(p^e_j/4t)^{1/(1-\alpha)}}{n^e_{j,1}}}_{\text{Duopoly effects}} & \text{if } \delta_j^*=0 \end{array}$$

The comparison between Eq. 4.5 and Eq. 4.6, and those of the Parker and Van Alstyne's model highlights two essential differences. First, the competition between platforms influences the optimal period of exclusivity. Competition does not influence the openness, but transportation costs and expectations influence it.

Nonetheless, as in the Parker and Van Alstyne's model, two equilibria are possible, but they depend on which region we pay attention to. An optimal positive δ is only possible if the total production at the second stage is larger than the difference in the outputs of developers on both platforms at the first stage plus the transportation costs.

This condition points out three key results. First, to set the period of exclusivity, platforms must pay attention to how much output developers may obtain on their platforms with respect to other platforms. If the production in the first period is large, they will set larger periods of time. In this case, platforms prefer to benefit from the royalties in this stage. On the other hand, if the production in the first period is small in comparison with the other platform, platforms prefer to reach sooner the second stage to profit from the royalties of that stage.

Second, the higher the transportation costs, the smaller δ, and the larger the period of exclusivity. Platforms prefer to set policies that maintain the rights of developers. Third, intuitively, the competition in the second stage creates the opposite effect. The higher the production of the competitor, the shorter the period of exclusivity. Platforms prefer to reach as soon as possible the second stage. Both developers and platforms can benefit from the higher royalties generated upon the cumulative innovation. Nonetheless, this condition is only true if $2\pi^d_{j,2}+\pi^d_{-j,2}>\Delta^1+6t$ holds, which I assume in that equilibrium. In any other case, the optimum would be to set a perpetual period of exclusivity. It is interesting that, under any circumstance, it is never optimal to force immediate openness of developers' rights. The same result was found by Parker and Van Alstyne (2017), and it seems to be robust in duopolistic frameworks.

With respect to the openness of the platforms, the transportation costs also influence the optimal openness. The higher the costs, the less open is the platform. The platforms prefer to focus on making profits out of users than to address heterogeneous developers. The same intuition applies to the role of users' expectations (n_e). The larger the expected users' demand is, the less open is the platform. On the other hand, the higher the apps prices (or their expectations), the more open is the platform. The intuition is straightforward, the higher those prices, the larger the royalties.

Proposition 1. *Competition with heterogeneous developers generates five essential results:*

1. *Competition leads to shorter periods of exclusivity*
2. *Transportation costs increase the period of exclusivity*
3. *The insights of Parker and Van Alstyne about the role of the production of the platform at the first and second stages still apply in duopolistic frameworks.*
4. *The higher the transportation costs or the expected users' demand, the less open are the platforms*
5. *The higher the apps prices (or their expectations), the more open are the platforms.*

Proof. See the appendix. □

[16]See `https://www.theregister.co.uk/2018/08/31/apple_privacy_policy/`

5 The role of taxes. Ad-valorem and unit taxes

In its current form, the DST is a revenue tax that would apply to marketplaces that facilitate transactions directly between users, and the transmission of collected user data, see KPMG (2018). Therefore, let's denote by τ^{vat} the ad-valorem tax, and by τ^{sp} the unit tax. When I introduce an ad-valorem tax, the Eq. 4.3 becomes

$$\max_{\sigma_j,\delta_j} \Pi_j = V_j n^e_{j,1}(1-\sigma_j) + \frac{1}{2}\frac{M^d_j p_j}{1+\tau^{vat}} \tag{5.1}$$

Respectively, when I introduce the unit tax, the Eq. 4.3 becomes

$$\max_{\sigma_j,\delta_j} \Pi_j = V_j n^e_{j,1}(1-\sigma_j) + \frac{1}{2}M^d_j\left(p_j - \tau^{sp}\right) \tag{5.2}$$

Solving Eq. 5.1, I find there is a unique pair $\langle \sigma^*, \delta^* \rangle$ that solves the maximization problem. In this case, the Eq. 4.5 and Eq. 4.6 become

$$\delta^*_j = \begin{cases} \frac{1}{3}\left[\frac{2\pi^d_{j,2}+\pi^d_{-j,2}-\Delta^1-6t}{\pi^d_{j,2}}\right] & \text{if } 2\pi^d_{j,2}+\pi^d_{-j,2} > \Delta^1+6t \\ 0 & \text{otherwise} \end{cases} \tag{5.3}$$

$$\sigma^*_j = \begin{cases} \frac{\alpha(\pi^d_{1,j}+\delta_j\alpha\pi^d_{2,j})}{4t(1+\tau^{vat})}\frac{v(1-\delta^*_j)}{2V_j n^e_{j,1}} & \text{if } \delta^*_j > 0 \\ \frac{(v\alpha p^e_j k/8t(1+\tau^{vat}))^{1/(1-\alpha)}}{V_j n^e_{j,1}} & \text{if } \delta^*_j = 0 \end{cases} \tag{5.4}$$

Ad-valorem taxes have no effect on the length of the exclusivity period, but they affect the openness decision. The higher the ad-valorem taxes, the smaller the openness. Note that the ad-valorem tax is levied on the revenues generated on developers. Therefore, in a similar way than Kind et al. (2008), platforms prefer to profit more from the other side (users). The overall effect is that platforms become more closed, and it is more difficult to innovate upon.
Instead of levying the tax on the developers' side, I could levy it on users side. In such case, Eq. 5.3 is unchanged, but Eq. 5.4 becomes

$$\sigma^*_j = \begin{cases} \frac{\alpha(\pi^d_{1,j}+\delta_j\alpha\pi^d_{2,j})}{4t}\frac{v(1-\delta^*_j)(1+\tau^{vat})}{2V_j n^e_{j,1}} & \text{if } \delta^*_j > 0 \\ \frac{(v\alpha p^e_j k(1+\tau^{vat})/8t)^{1/(1-\alpha)}}{V_j n^e_{j,1}} & \text{if } \delta^*_j = 0 \end{cases} \tag{5.5}$$

Platforms become more open. The intuition is analogous to the previous case, given the taxation, platforms prefer to be more open because in that way they can profit from developers' innovations and the royalties generated.
On the other hand, if we solve Eq. 5.2, there is a unique pair $\langle \sigma^*, \delta^* \rangle$ that solves the maximization problem too. In this case, the Eq. 4.5 and Eq. 4.6 become

$$\delta^*_j = \begin{cases} \frac{1}{3}\left[\frac{(v-\tau^{sp})(2\pi^d_{j,2}+\pi^d_{-j,2})-\Delta^1-6tv}{v\pi^d_{j,2}}\right] & \text{if } (v-\tau^{sp})2\pi^d_{j,2}+\pi^d_{-j,2} > \Delta^1+6tv \\ 0 & \text{otherwise} \end{cases} \tag{5.6}$$

$$\sigma^*_j = \begin{cases} \frac{\alpha(\pi^d_{1,j}+\delta_j\alpha\pi^d_{2,j})}{4t}\frac{v(1-\delta^*_j)-\tau^{sp}}{2V_j n^e_{j,1}} & \text{if } \delta^*_j > 0 \\ \frac{((v-\tau^{sp})\alpha p^e_j k/8t)^{1/(1-\alpha)}}{V_j n^e_{j,1}} & \text{if } \delta^*_j = 0 \end{cases} \tag{5.7}$$

Unit taxes increase the period of exclusivity. Nonetheless, it also changes the threshold at which platform set $\delta_j = 0$, that implies that platforms are more willing to not to absorb the developers' rights with such taxes. Unit taxes also reduce the openness of platforms. Therefore, platforms are less open to innovators, which is against one of the pillars of the digital strategy of the EU[17]. In fact, the reduction of openness as a consequence of this tax can be more drastic than with ad-valorem taxes because of the change in the threshold of δ.
Instead of levying the unit tax on the developer's side, I can levy it on the users' side. In this case, Eq. 5.3 is unchanged, but Eq. 5.4 becomes

$$\sigma^*_j = \begin{cases} \frac{\alpha(\pi^d_{1,j}+\delta_j\alpha\pi^d_{2,j})}{4t}\frac{v(1-\delta^*_j)}{2V_j n^e_{j,1}-\tau^{sp}} & \text{if } \delta^*_j > 0 \\ \frac{(vV_j\alpha p^e_j k/8t(V_j n^e_{j,1}-\tau^{sp}))^{1/(1-\alpha)}}{V_j} & \text{if } \delta^*_j = 0 \end{cases} \tag{5.8}$$

[17] See `https://ec.europa.eu/digital-single-market/en/open-innovation-20`

In this case, the unit tax increases the openness of the platforms. This increase is more important when platforms set $\delta = 0$ because the unit tax does not only lead to a smaller denominator in Eq. 5.8, but it also increases the numerator[18].
These results point out that taxation modifies how platforms make profits, as predicted by Kind et al. (2008). However, this time is not the mechanism of prices, but the IP policies. Therefore, the consequences of the DST could be large than initially thought because its impact it is not limited to prices, which makes even more difficult to evaluate its impact in terms of welfare.

Proposition 2. *If royalties or developers' revenues are taxed, platforms become less open and the set of tools available to innovate is smaller. If taxes are levied on revenues from users, platforms tend to set more open policies.*

Proof. See the appendix. □

Taxation of digital platforms modifies the optimal intellectual property policies of platforms. Therefore, I expect a lower lever of innovation in the European ecosystem if the DST comes into force in 2020 in its current form. It does not matter if revenues are generated either by monetization of data or advertising. What really matters is whether or not the European Union levy the tax on the revenues from those who generate innovations, such as developers.

6 Use of both platforms at the same time. Multihoming developers

If we compare Google Play and the App Store today, we can observe that many apps are available on both platforms, such as Facebook, Airbnb, Tinder or Uber. This is an essential characteristic of digital markets. Users normally have only one kind of device (an Android smartphone or an iPhone), and developers normally make their apps available on both platforms. In other words, developers multihome.
In this section, I relax the singlehoming assumption, and I consider that developers can multihome. Therefore, the utility of a developer who multihomes is

$$U^d_{mh} = \frac{\pi_{1,j} + \pi_{1,-j} + \delta_j(\pi_{2,j} + \pi_{2,-j})}{2} - t \tag{6.1}$$

Therefore, platforms address two different kinds of developers. Those who multihome, and those who singlehome. That implies two immediate consequences. First, demand will be higher than one, because there are developers that demand both platforms. Second, demands are formed by two different populations of developers[19].

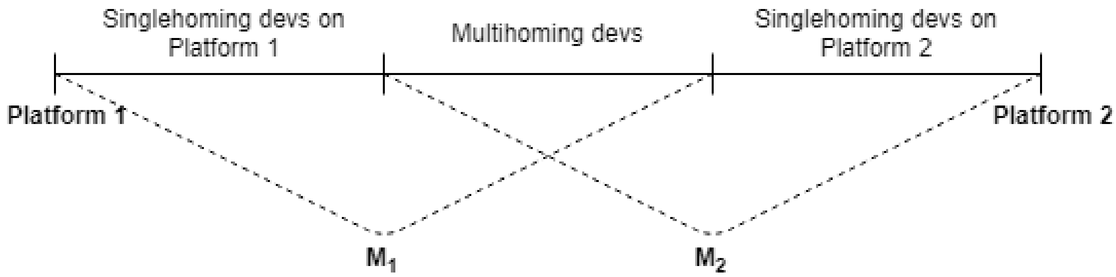

Figure 1: Market structure with multihoming developers

To characterize the demands addressed to each platform, I to identify the marginal users between singlehoming and multihoming on both platforms. Formally, $U^d_{mh} \leq U^d_j$ and $U^d_{mh} \leq U^d_{-j}$. Solving these two expression,

$$\begin{aligned} x_1 &\leq 1 - \tfrac{\pi_{1,-j} + \delta_{-j}\pi_{2,-j}}{2t} \\ x_2 &\geq \tfrac{\pi_{1,j} + \delta_j \pi_{2,j}}{2t} \end{aligned}$$

Using these expressions, I can directly derive the demands

$$M^d_j = m^d_{sh} + m^d_{mh} = \frac{\pi_{1,j} + \delta_j \pi_{2,j}}{2t} \tag{6.2}$$

Note that $M_1 + M_2 > 1$ if $m^d_{mh} > 0$, which I assume. Then, I solve Eq. 4.3 using this new demand function, and I find that the expression of the interior solution of σ does not change too much. In fact, the only change

[18] In this case, we have V_j in the numerator.
[19] A similar approach can be found in Choi et al. (2010).

is the constant in the denominator that it is smaller in this case. Therefore, multihoming influences positively the degree of openness by a constant factor.
On the other hand, in the case of δ, I find that the interior solution is the same than in Parker and Van Alstyne (2017). The presence of multihoming developers mitigates the competition between platforms. They behave like monopolies with respect to developers when setting the exclusivity period.

$$\delta_j^* = \begin{cases} \frac{1}{2}\left[1 - \frac{y_1}{y_2}\right] & \text{if } y_2 > y_1 \\ 0 & \text{otherwise} \end{cases} \tag{6.3}$$

In this situation, it is also interesting to address the impact of taxes. Solving Eq. 5.1 with the demand derived in this section, I find that δ is not influenced by ad-valorem taxes, and because σ behaves as in the previous section, there are no changes in the effect of ad-valorem taxes. On the other hand, solving Eq. 5.2 with multihoming developers, I find that the Eq. 5.6 becomes

$$\delta_j^* = \begin{cases} \frac{1}{2}\left[1 - \frac{y_1}{y_2} - \frac{\tau^{sp}}{v}\right] & \text{if } y_2 > y_1 \\ 0 & \text{otherwise} \end{cases} \tag{6.4}$$

On the other hand, there are no big changes in σ as a consequence of multihoming, and therefore, the effect of taxation will be the same than in the singlehoming model.

Proposition 3. *When there are multihoming developers, platforms set their monopolistic exclusivity period. On the other hand, the effects of taxation are not qualitatively different from those in the singlehoming case.*

7 Extension: Dynamic decisions with limited knowledge: The Gradient Rule

Platforms are dynamic entities that are changing their policies from time to time. These changes are not radical, no platform changes from open source to private source code in one year, and in the next one, it comes back again to open source. Normally, the changes are marginal, in the sense of making small changes in their IP policies because the knowledge of the market is limited, and it is difficult to predict what will happen if they move to the other extreme of the market. Therefore, it makes sense to analyze how platforms behave when they have limited knowledge of the demand function.
I assume that platforms use the gradient mechanism. Intuitively, they pay attention to the variation in their profits because of changes in their IP policies. A positive (negative) variation of profits induces the platforms to change their policies in the same (opposite) direction than the previous period. No changes will occur if profits are constant. This mechanism can be represented as follows

$$\begin{aligned} \delta_j(t+1) &= \delta_j(t) + \lambda_\delta \frac{\partial \Pi_j}{\partial \delta_j(t)}, t = 0, 1, 2, ... \\ \sigma_j(t+1) &= \sigma_j(t) + \lambda_\sigma \frac{\partial \Pi_j}{\partial \sigma_j(t)}, t = 0, 1, 2, ... \end{aligned} \tag{7.1}$$

Where λ is the speed of adjustment to misalignments. Substituting the first-order conditions (FOC) of Eq. 5.1 and Eq. 5.2 in Eq. 7.1, we obtain a one-dimensional nonlinear difference equation that represents the behavior of σ, and a two-dimensional nonlinear difference system that represents the behavior of δ[20].

$$\delta_j(t+1) = \delta_j(t) + \lambda_\delta \left[\frac{[1-\delta_j(t)]v\pi_{2,j}^d}{8t} - v\left(1/4 + \frac{\pi_{1,j}^d + \delta_j(t)\pi_{2,j}^d - \pi_{1,-j}^d - \delta_{-j}(t)\pi_{2,-j}^d}{8t}\right)\right] \tag{7.2}$$

$$\sigma_j(t+1) = \sigma_j(t) + \lambda_\sigma \left[-V_j n_{j,1}^e + \frac{(1-\delta)v(\alpha\sigma_j(t)^{\alpha-1}V_j^\alpha p_j^e k + p_j^e m_{e,j}^\alpha \alpha^2 \delta_j k^{1+\alpha}\sigma_j(t)^{\alpha^2-1}V_j^{\alpha^2})}{8t}\right] \tag{7.3}$$

7.1 Equilibrium analysis

If

$$f(\sigma_j) = \sigma_j + \lambda_\sigma \left[-V_j n_{j,1}^e + \frac{(1-\delta)v(\alpha\sigma_j^{\alpha-1}V_j^\alpha p_j^e k + p_j^e m_{e,j}^\alpha \alpha^2 \delta_j k^{1+\alpha}\sigma_j^{\alpha^2-1}V_j^{\alpha^2})}{8t}\right] \tag{7.4}$$

[20] It may be argued that I should have a two-dimensional system when choosing the level of openness because there are two platforms. However, given that the decisions of openness (σ) are independent, that is not the case. Additionally, when choosing the time of exclusivity (δ), platforms do not take into account their own openness (see Eq. 4.5), so I analyze both decisions independently.

$$f(\delta_j) = \delta_j + \lambda_\delta \left[\frac{[1-\delta_j] v \pi^d_{2,j}}{8t} - v \left(1/4 + \frac{\pi^d_{1,j} + \delta_j(t)\pi^d_{2,j} - \pi^d_{1,-j} - \delta_{-j}\pi^d_{2,-j}}{8t} \right) \right] \tag{7.5}$$

The fixed points of Eq. 7.2 and Eq. 7.3 are solutions of the equations $f(\delta_j) = \delta_j$ and $f(\sigma_j) = \sigma_j$. Therefore, the fixed points are the solutions derived from the FOC of Eq. 4.3. Given:

$$\frac{\partial f(\delta)}{\partial \delta} = \begin{cases} \frac{\partial f(\delta_j)}{\partial \delta_j} = 1 + \lambda_\delta \frac{\partial^2 \Pi_j(\delta_j, \delta_{-j})}{\partial \delta_j^2} \\ \frac{\partial f(\delta_{-j})}{\partial \delta_{-j}} = 1 + \lambda_\delta \frac{\partial^2 \Pi_j(\delta_j, \delta_{-j})}{\partial \delta_{-j}^2} \end{cases} \tag{7.6}$$

$$\frac{\partial f(\sigma_j)}{\partial \sigma_j} = 1 + \lambda_\sigma \frac{\partial^2 \Pi_j}{\partial \sigma_j^2} \tag{7.7}$$

The steady states would be locally stable if

$$\begin{gathered} 0 < \lambda_\delta < \frac{16t}{\pi_{2,j} + \pi_{2,-j} + (\pi^2_{2,j} + \pi^2_{2,-j} - \pi^2_{2,j}\pi^2_{2,-j})^{1/2}} \\ 0 < \lambda_\sigma < \frac{2}{-\Pi''_j(\sigma_j)} \end{gathered} \tag{7.8}$$

It follows that Eq. 7.2 and Eq. 7.3 have two steady states that correspond to the equilibrium points defined in Eq. 4.5 and Eq. 4.6. Those points are locally stable if the conditions defined in Eq. 7.8 hold.
These results point out that with small changes in the IP policies, it will be possible to converge to the optimal pair $\langle \sigma^*, \delta^* \rangle$ even when the platforms do not know all the demand function. On the other hand, overreactions (λ_σ and λ_δ values that violate the Eq. 7.8 conditions) may lead to a suboptimal solutions. As a summary, it is interesting to point out that the equilibria previously defined are still valid even in the presence of limited knowledge.

Proposition 4. *The optimal pair $\langle \sigma^*, \delta^* \rangle$ when the knowledge of platforms is limited is the same than the optimal pair when platforms have full information. Even when the information about the demand is limited, small changes in the IP policies may lead to convergence to the optimal set of policies over time.*

Proof. See the appendix. □

8 Conclusions. The future of the digital taxation

Under the idea that there is a disconnect – or 'mismatch' - between where value is created and where taxes are paid, the European Commission has proposed a reform of corporate tax rules that will come into force on January 2020[21]. As KPMG (2018) points out, the Digital Service Tax (DST) should be seen in the context of fighting against base erosion and profit shifting (BEPS) but also, it has a clear objective of collecting revenues from the digital markets, which are becoming more and more relevant each day.
If we put aside the fiscal revenues for a moment, we know from Parker and Van Alstyne (2017) that a social planner prefers open platforms and short exclusivity periods. My results show that taxation leads to an equilibrium far away from the social planner solution. Nonetheless, there is a missing point. Taxation also influences the price structure.
Kind et al. (2008) and Kind and Koethenbuerger (2018) show that taxing one side may shift the business model towards "other sides". Intuitively, taxing the added-value created on Europe may incentive platforms to generate such value somewhere else. Either in another geographical market or in another non-taxed market within Europe. Additionally, Bourreau et al. (2018) show that a tax on data may fail to increase fiscal revenues. All these results raise concerns about whether or not to tax developers' activities. Taxing the developers' activities reduces the openness of platforms, and therefore, the potential added-value that could be generated upon those platforms. At the same time, taxes influence the price structure, which is modified to compensate the taxation. Depending on the kind of taxation chosen, fiscal revenues may not increase as expected, or even worse, a tax designed to be levied on developers' activities may be paid by inelastic users in the end.
In this work, I extend the Parker and Van Alstyne's model to address the impact of competition and taxes on the openness of platforms and on the exclusivity period awarded to developers to exploit their innovations. Although research on the impact of taxes on digital platforms has been going on for more than a decade, the impact of taxes on strategic decisions such as intellectual property policies has not been addressed up to the date.
I find that competition shorten the exclusivity period, and therefore, developers keep for longer periods their rights over their innovations, but it has no impact on the openness. I also find that larger differentiation implies larger exclusivity periods, but platforms less open. I confirm a previous result highlighted by Parker and

[21] `https://ec.europa.eu/taxation_customs/business/company-tax/fair-taxation-digital-economy_en`

Van Alstyne (2017), it is never optimal to force immediate openness of developers' rights.
When I introduce taxes, I observe that ad-valorem taxes levied on the royalties paid by developers have no effect on the length of the exclusivity period, but the higher the ad-valorem taxes, the lower the openness. The opposite is true if such tax is levied on users. In the case of unit taxes levied on revenues from developers, they increase the period of exclusivity and reduce the openness. However, if such taxes are levied on revenues from users, platforms become more open. I prove that these insights are also robust with multihoming developers, and I also show that multihoming leads to shorter exclusivity periods because of the mitigation of competition. I conclude by analyzing the case of platforms with limited knowledge about their demands, and I prove that our previous results also hold in this case.
Taxation of digital platforms modifies the optimal intellectual property policies of platforms. If we tax developers activities, platforms become less open, and the set of tools available to developers is smaller. This conclusion also applies to advertisers. Taxing revenues obtained from advertisers would reduce platforms openness, and innovation in that sector would be lower. Therefore, I expect a lower level of openness and innovation in the European ecosystem if the DST comes into force.

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Appendices

A Optimal solutions for δ and σ. Proof Proposition 1

Taking the Eq. 4.3, the first-order conditions on platforms profits with respect to δ_j are

$$\frac{\partial \Pi_j}{\partial \delta_j} = \frac{[1-\delta_j]v\pi^d_{2,j}}{8t} - v\left(1/4 + \frac{\pi^d_{1,j} + \delta_j\pi^d_{2,j} - \pi^d_{1,-j} - \delta_{-j}\pi^d_{2,-j}}{8t}\right) = 0 \tag{A.1}$$

If I rearrange terms in Eq. A.1, I arrive at the first case of Eq. 4.5. Note that the second-order conditions are $\frac{\partial^2 \Pi_j}{\partial \delta_j^2} = \frac{-k^{1+\alpha} v m^{\alpha}_{e,j} p^e_j (\sigma_j V_j)^{\alpha^2}}{4t} < 0$, it implies that Π_j is concave in δ_j, and by solving the first-order conditions, I obtain the global maximum. Also, note that this interior solution only exists if $2\pi^d_{j,2} + \pi^d_{-j,2} > \Delta^1 + 6t$. This expression can be easily derived from Eq. 4.5. Otherwise, $\delta_j = 0$, the corner solution.
Taking the Eq. 4.3, the first-order conditions on platforms profits with respect to σ_j are

$$\frac{\partial \Pi_j}{\partial \sigma_j} = -V_j n_e + \frac{(1-\delta)v(\alpha\sigma_j^{\alpha-1}V_j^{\alpha}p^e_j k + p^e_j m^{\alpha}_{e,j}\alpha^2\delta_j k^{1+\alpha}\sigma_j^{\alpha^2-1}V_j^{\alpha^2})}{8t} = 0 \tag{A.2}$$

Again, if I rearrange terms in Eq. A.2, I arrive at the first case of Eq. 4.6. In this case, the second-order conditions are $\frac{\partial^2 \Pi_j}{\partial \sigma_j^2} = \frac{(1-\delta_j)v\left((\alpha-1)\alpha k p^e_j \sigma_j^{\alpha-2}V_j^{\alpha} + \alpha^2(\alpha^2-1)\delta_j k^{1+\alpha} m^{\alpha}_{e,j} p^e_j \sigma_j^{\alpha^2-2}V_j^{\alpha^2}\right)}{8t} < 0$, it implies that Π_j is concave in σ_j, and by solving the first-order conditions, I obtain the global maximum. Also, note that this interior solution only exists if $\delta_j > 0$. Otherwise, I only have to substitute $\delta_j = 0$ in Eq. A.2, and σ would be equal to the second case of Eq. 4.6, the corner solution.
Lastly, Eq. 4.6 shows that σ depends on $\pi_{i,j}$, $i = 1, 2$ that also depends on σ. This situation may raise concerns about the uniqueness of the solution, but the proof of it is similar to the one in Parker and Van Alstyne (2017). Nonetheless, it is a little bit more tedious. For simplicity's sake, I omit it here, but it is available upon request.

B Optimal solutions for δ and σ. Proof Proposition 2

Taking the Eq. 5.1, the first-order conditions on platforms profits with respect to δ_j are the same than before. Therefore, the previous proof applies here. On the other hand, if we take Eq. 5.2, the first-order conditions on platforms profits with respect to σ_j are

$$\frac{\partial \Pi_j}{\partial \sigma_j} = -V_j n_e + \frac{(1-\delta)v(\alpha\sigma_j^{\alpha-1}V_j^{\alpha}p^e_j k + p^e_j m^{\alpha}_{e,j}\alpha^2\delta_j k^{1+\alpha}\sigma_j^{\alpha^2-1}V_j^{\alpha^2})}{8t(1+\tau^{vat})} = 0 \tag{B.1}$$

If I rearrange terms, I arrive at the first expression of Eq. 5.4. In this case, the second-order conditions also verify that the interior equilibrium is the global maximum. Nonetheless, it also exists a corner solution that is reached when $\delta_j = 0$, as in the previous case. Lastly, if I take Eq. 5.2, the first-order conditions on platforms profits with respect to δ_j are

$$\frac{\partial \Pi_j}{\partial \delta_j} = \frac{([1-\delta_j]v - \tau^{sp})\pi^d_{2,j}}{8t} - v\left(1/4 + \frac{\pi^d_{1,j} + \delta_j\pi^d_{2,j} - \pi^d_{1,-j} - \delta_{-j}\pi^d_{2,-j}}{8t}\right) = 0 \tag{B.2}$$

If I rearrange terms, I arrive at the first expression of Eq. 5.6. In this case, the second-order conditions also verify that the interior equilibrium is the global maximum. Nonetheless, it also exists a corner solution that is reached when $(v-\tau^{sp})2\pi^d_{j,2} + \pi^d_{-j,2} > \Delta^1 + 6tv$. Lastly, if I take Eq. 5.2, the first-order conditions on platforms profits with respect to σ_j are

$$\frac{\partial \Pi_j}{\partial \sigma_j} = -V_j n_e + \frac{((1-\delta)v - \tau^{sp})(\alpha\sigma_j^{\alpha-1}V_j^{\alpha}p^e_j k + p^e_j m^{\alpha}_{e,j}\alpha^2\delta_j k^{1+\alpha}\sigma_j^{\alpha^2-1}V_j^{\alpha^2})}{8t} = 0 \tag{B.3}$$

If I rearrange terms, I arrive at the first expression of Eq. 5.7. The second-order conditions also verify that the interior equilibrium is the global maximum, and it also exists the corner solution that is reached when $\delta_j = 0$ as in the previous cases.

C Stability of the bidimensional dynamic system. Proof Proposition 4

Taking the Eq. 7.2, I have a bidimensional nonlinear difference system, $\delta(\delta_j, \delta_{-j})$, which steady states are the equilibrium points previously defined. Such steady states derived from Eq. 7.6 are locally stable if the modulus of the eigenvalues of the Jacobian matrix are strictly smaller than 1. The Jacobian matrix of the system is as follows

$$J(\delta_j, \delta_{-j}) = \begin{pmatrix} 1 + \lambda_\delta \frac{-\pi_{2,j}}{4t} & \lambda_\delta \frac{\pi_{2,-j}}{8t} \\ \lambda_\delta \frac{\pi_{2,j}}{8t} & 1 + \lambda_\delta \frac{-\pi_{2,-j}}{4t} \end{pmatrix} \tag{C.1}$$

If I compute the eigenvalues, $det(A - I\lambda) = 0$, I have two negative roots ($\lambda_1, \lambda_2 < 0$). Therefore, the following conditions must hold to be locally stable

$$0 < \lambda_\delta < \frac{16t}{\pi_{2,j} + \pi_{2,-j} + (\pi_{2,j}^2 + \pi_{2,-j}^2 - \pi_{2,j}^2\pi_{2,-j}^2)^{1/2}}$$

If I analyze the same system but under the multihoming scheme, the dynamic of σ does not change because the multihoming does not affect how the σ is chosen. Nonetheless, multihoming influences the decision on δ. In this case, the Jacobian matrix becomes

$$J(\delta_j, \delta_{-j}) = \begin{pmatrix} 1 + \lambda_\delta \frac{-\pi_{2,j}}{2t} & 0 \\ 0 & 1 + \lambda_\delta \frac{-\pi_{2,-j}}{2t} \end{pmatrix} \tag{C.2}$$

Therefore, the eigenvalues can be derived directly, and the following condition must hold

$$0 < \lambda_\delta < \frac{4t}{\pi_{2,j}}$$

On the other hand, given that the dynamic of σ is represented by a one-dimensional nonlinear difference equation, the derivation of the stability condition is trivial.