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A Model of Market Making with Heterogeneous Speculators

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A Model of Market Making with Heterogeneous Speculators

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Abstract

I introduce an optimizing monopolistic market maker in an otherwise standard setting *a la* Brock and Hommes (1998) (BH98). The market maker manages her inventory of a zero yielding asset, such as foreign currency, and can earn profits from trading, taking advantage of her knowledge of speculators' demand. The resulting dynamic behavior is qualitatively identical to the one described in BH98, showing that the results of the latter are independent from the institutional framework of the market. At the same time I show that the market maker has conflicting effects. She acts as a stabilizer when she allows for market imbalances, while she acts as a destabilizer when she manages aggressively her inventories and when she trades actively, both if she acts as fundamentalist or if she is a strong extrapolator. Indeed the more stable institutional framework is one in which market makers are inventory neutral and don't trade actively but, even in this case, the typical complex behavior of BH98 occurs.

Keywords: Asset pricing model, heterogeneous beliefs, market architecture, market making, foreign exchange market.

JEL codes: G12, D84, D42, C62, F31

1 Introduction

Models of financial markets with heterogeneous bounded rational speculators generally come in two flavors. The seminal model of Brock and Hommes (1998) makes the standard assumption of market clearing. Other models assume instead the existence of a market maker, which accumulates inventories and adjusts the price in the same direction of net demand following a simple linear rule (Day and Huang, 1990; Lux, 1995).

The market maker hypothesis is widely employed in analyses of the FX market, since it is considered a better description of the actual price adjustment mechanism, given the pivotal role of dealers in what is a substantially decentralized market (Westerhoff, 2009). The standard linear adjustment rule, which is generally used in these models, might lead to large inventory unbalances. This implication is inconvenient, since the empirical evidence shows that FX dealers actively manage their inventories in order to end the trading day on a balanced position (Manaster and Mann, 1996; Bjønnes and Rime, 2005). Westerhoff (2003) incorporates inventory management in the price adjustment rule, showing that a more aggressive control of inventories makes the market more volatile and less stable. Carraro and Ricchiuti (2015) and Zhu *et al.* (2009), following Madhavan and Smidt (1991), extend this framework by considering market makers who take speculative positions by adjusting their inventories towards a given exogenous target.

This paper presents a model which incorporates a more sophisticated representation of the behavior of the market maker than the current literature on HAM (Heterogeneous Agents Models). In particular, I suppose that there are two types of speculators (fundamentalists and chartists) who submit their trades to a monopolistic market maker who is an optimizing bounded rational agent. I further suppose that the market maker knows the optimal demand of speculators, in accordance with the fact that "knowing the market" is the most important asset for FX dealers (see Sec. 2). In a first version of "pure" market making, her only source of profits comes from market making itself, thus she sets her optimal price solving the trade off between higher income per traded unit and decreasing marginal inventory costs on the one hand, and lower net demand on the other. The second version includes an "activist" market maker who is allowed to trade with bounded rational expectations.

This paper relates to previous HAMs, and in particular to Hommes *et al.* (2005), who extend the BH98 framework to a market maker scenario, using a linear adjustment rule, and prove that the dynamic behavior of the system is pretty similar to BH98. We confirm this result in our framework while, on the other hand, we show that the market maker has conflicting effects on the stability of the market. From this perspective, the description of the market maker in this paper is closer to the one of Zhu *et al.* (2009) and Carraro and Ricchiuti (2015), who model explicitly the role of the market maker as an active investor and also underline that the market maker destabilizes the market when she manages her inventory. On the other hand, these models don't derive the pricing rule from an optimizing behavior of the market maker as I do in this paper. Instead they introduce an ad hoc pricing mechanism.

Zhu *et al.* (2009) and Carraro and Ricchiuti (2015) provide analytical results for fixed proportions of speculators, while the results of the following sections are derived using the heuristic switching mechanism of BH98 which allows these proportions to evolve endogenously. Anufriev and Panchenko (2009) provide through simulations a comparison of different market protocols (Walrasian auctioneer, market maker, batch auction and order book) allowing for the endogenous evolution of the proportions of speculators. They show that, no matter which type of market clearing is used, two different regimes with completely different dynamical properties occur depending on the value of the intensity of choice, and that the trading protocol strongly affects the critical value of the intensity of choice. Since I derive the market clearing case of BH98 as a special case of the model presented in this paper, it becomes possible to prove analytically that the trading protocol affects the critical values of the intensity of choice, which separate the two different dynamic regimes.

The remaining of this paper is organized as follows. In sec. 2 I relate the hypotheses adopted in this paper to the literature on the microstructure of FX markets. In sec. 3 I present the model with a "pure" market maker, and in sec. 4 the model with an "activist" market maker. Sec. 5 concludes.

2 Related Literature

According to the market microstructure literature, net order flows are strong predictors of exchange rate movements. Foreign exchange dealers believe that private information is crucial to operate on the exchange rate market and that trading flows collected from customers aggregate dispersed information (King *et al.*, 2013). On the other hand, the possibility of private information in the FX market has been questioned since the fundamental value of a currency is determined by macroeconomic information (e.g. regarding interest rates or inflation) which is publicly available. Indeed the empirical evidence of an informational advantage on the FX market relates to specific situations such as central bank intervention on the currency market (Peiers, 1997). Instead, insider models like Kyle (1985) or Glosten and Milgrom (1985) draw their inspiration from the stock market, where a privileged access to information is more likely to occur. According to Glosten and Milgrom (1985), market spreads exists because market makers face an adverse selection problem and set the spread in order to recoup with liquidity traders the losses incurred when trading with informed counterparts. One major problem with these models is that market makers earn zero profits, thus they would have no incentive to perform their role. Their purpose is to explain the bid-ask spread as an informational phenomenon which would persist even when market makers make no profits.

But what is the source of private information on the FX market if fundamentals are common knowledge? One possible answer is that private information that is most valuable does not concern fundamentals. One source of non fundamental information, according to King *et al.* (2013), stems from demand and supply themselves. If they have only finite elasticity, i.e. if the liquidity of markets is limited, market makers can leverage on their role to make profits. Following this line, Cai *et al.* (2001) show with high frequency data that customer order flows have an influence on rates distinct from macroeconomic announcements. Thus the anticipation of public announcements is not the only potential source of private information on the foreign exchange market.

This line of thought leads us to the next problem, namely: who is informed on the FX market? Using a detailed breakdown of customer typologies, Osler and Vandrovych (2009) show that only leveraged investors bring information to the market. All other types of customers appear to be uninformed, while banks themselves appear to be better informed than their customers. Specifically, the price impact of bank trades remains strong for up to one week, while the price impact of leveraged-investors loses significance after six hours.

These result confirm the common view among FX dealers that big banks are better informed because they trade with the biggest customers (Cheung and Chinn, 2001). The intuition is that banks, by servicing their customers, collect dispersed information from the market which they put to a good use for their own trades. This view is supported also by the empirical evidence that spreads are narrower for financial customers and for larger trades (Osler *et al.*, 2011). This stylized fact is inconsistent with adverse selection models like Kyle (1985) or Glosten and Milgrom (1985), according to which market makers should charge larger spreads to the most informed traders and on larger trades, which are more likely to be originated from informed counterparts. To motivate this pricing choice we must refer to factors like market power and strategic dealing. According to this view, FX dealers choose to attract large order flows by selectively setting competitive quotes in order to understand promptly the direction of the market.

The opportunity of profits for dealers arise because FX is a two-tier market and dealers may use the information gathered with customers in the first tier to profit from interdealer trades in the second tier. For instance, the results of Osler *et al.* (2011) show that dealers are more likely to trade aggressively on the interdealer market after trades with informed counterparts. We might wonder what would be the optimal pricing and trading strategy for a dealer under these circumstances. Empirical evidence shows that FX dealers unload inventories quickly and do no adjust their quotes following an inventory unbalance (Bjønnes and Rime, 2005). Current theoretical models distinguish between problems of inventory management and of adverse selection. In inventory-based models, risk-averse market makers adjust prices to induce a trade in a certain direction (Ho and Stoll, 1981; Huang and Stoll, 1997; Madhavan and Smidt, 1991). For instance, a FX dealer with a long position in USD may reduce his ask to induce a purchase of USD by his counterparts. Information-based models predict instead that, when a market maker receives a trade initiative from a counterpart she deems as informed, she will raise or lower her price quote conditioned on whether the initiative ends with a "Buy" or a "Sell".

Both categories of models agree on the prediction that buyer-initiated trades will make the market maker raise prices, while seller-initiated trades will have the opposite effect. But both categories are at odds with the empirical evidence mentioned above, which instead points to the idea that FX dealers leave the price unchanged and profit from the future movement of price by trading as quickly as possible in the same direction of their incoming trades. An optimal trading strategy of this sort is derived in the widely considered model of Evans and Lyons (2002) and is linked to the so called "hot potato trading" on the interdealer market (Lyons, 1997), i.e. the passing through of undesired inventory positions among FX dealers. Consistently with this view, Manaster and Mann (1996) conclude that market makers have informational advantages that enable them to adjust their inventory in anticipation of favorable price movements and Bjønnes and Rime (2005) find evidence that FX dealers engage in information-based speculation.

To sum up, evidence collected from FX market data shows that market making is a valuable source of information for taking speculative positions and that large financial entities like banks seek actively and employ extensively this sort of information. In this paper I try to incorporate this evidence by making the admittedly simplifying assumption that the market maker has perfect knowledge of customer demand.

This assumption do not exclude that market imbalances affect prices. In order book markets like the interdealer FX market, an excess of trading in one direction impacts price since liquidity is not unlimited, and price variations on the wholesale market will quickly transmit to prices charged to customers. In the model of Evans and Lyons (2002) noise traders or 'profit takers' enter to clear the market and allow FX dealers to profit from end-of-day trades after a price adjustment occurs on the interdealer market which is transmitted to the retail market. In this paper, I suppose the market maker is able to satisfy the net demand coming from customers by adjusting in advance her inventory at the current price, i.e. *before* she announces the new optimal price to her customers and trading with speculators begins.

3 Model with a "pure" market maker

I assume that the market maker is a monopolist which trades a zero yielding asset with a large number of different types of speculators whose weights on the market evolve endogenously. In each period, she fixes an optimal price, taking into account a quadratic cost of inventory maintenance. I further assume that the market maker knows the optimal demand of each type of speculator and that she employs this information when she solves her objective. Then I suppose that the market is liquid enough to allow the market maker to adjust in advance her inventory at the current market price, in order to match the projected orders of speculators. Once the new price is revealed to speculators, the latter trade according to their optimal demand in such a way that, at the end of the period, the net variation of the inventory position of the market maker is zero.

The timeline of events in each period of the model is pictured in Fig. 1. We remark that the assumption on market liquidity is analogous to the one made by Evans and Lyons (2002), who suppose that FX dealers might trade on the wholesale market *before* the price adjustment on the retail market occurs. It is also in line with evidence from the FX market (see Sec. 2) and in particular with the practice of "hot potato" trading

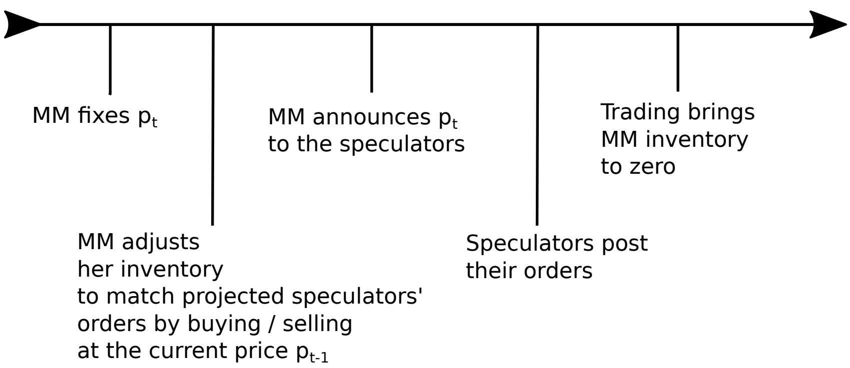


Figure 1: Timeline of events occuring within a single period of the model.

which allows FX dealers to profit from retail trading.

Recalling that we are considering the case of a zero yielding asset, the wealth of the market maker evolves according to the following equation:

$$W_{d,t} = R\,W_{d,t-1} + (p_t - p_{t-1}) \sum_{h=0}^{H-1} n_{h,t} z_{h,t} - \omega \left(\sum_{h=0}^{H-1} n_{h,t} z_{h,t} \right)^2 \tag{1}$$

where $z_{h,t}$ is the demand of speculators of type h and $n_{h,t} = \frac{e^{\beta U_{h,t-1}}}{Z_t}$ is the fraction of speculators of type h. In this formula Z_t is a normalization factor and

$$U_{h,t-1} = (p_{t-1} - Rp_{t-2})\, z_{h,t-2} - C_h \tag{2}$$

is the fitness of the trading strategy h, where C_h is a type specific fixed cost. The market maker adjusts the spread $p_t - p_{t-1}$ she charges to speculators in order to maximize her wealth at t. At the same time, she bears an increasing marginal cost for carrying the inventory, which is equal to the projected demand of speculators and is entirely liquidated by the end of the period, as explained above, since the market maker has a perfect knowledge of market demand.

The wealth of speculators of type h evolves in a standard way:

$$W_{h,t} = R\,W_{h,t-1} + (p_t - Rp_{t-1})\, z_{h,t-1} \tag{3}$$

Both the speculators and the market maket act to maximize their wealth. Speculators are mean-variance maximizers, since their future wealth is uncertain. Indeed their wealth at t is determined by their net demand at $t-1$ and they ignore the future market price at t when taking their decision at $t-1$. The market maker instead is not subject to uncertainty. Moreover she is aware of the optimal demand of speculators $z^*_{h,t}$ when she optimizes. This assumption is consistent with the monopoly position which market makers have, especially on markets where the largest part of transations are over the counter, like the FX market (see sec. 2).

It is convenient to rewrite both problems in terms of deviations from a fundamental value $x_t = p_t - p^*_t$. In the case of speculators it is possible to do so by assuming the standard pricing relationship $p^*_t = E_t[p^*_{t+1}]/R$. The expectation of speculators of type h on x_t is $E_h\left[x_{t+1}\right] = b_h + g_h x_{t-1}$. The objectives are respectively

- Dealers:
$$\max_{x_t}\left\{(x_t - x_{t-1})\sum_{h=0}^{H-1} n_{h,t} z^*_{h,t} - \omega\left(\sum_{h=0}^{H-1} n_{h,t} z^*_{h,t}\right)^2\right\} \tag{4}$$

- Speculators of type h:

$$\max_{z_{h,t}}\left\{\left(b_h + g_h x_{t-1} - R x_t\right) z_{h,t} - \frac{z^2_{h,t}}{2D}\right\} \tag{5}$$

The optimal demand of speculators of type h is the standard one:

$$z^*_{h,t} = D\left(b_h + g_h x_{t-1} - R x_t\right) \tag{6}$$

Substituting eq. (6) in (4) and deriving we obtain the FOC for market maker which can be solved for x_t:

$$x_t = \frac{1}{DR\omega + 1}\left\{\frac{x_{t-1}}{2} + \left(D\omega + \frac{1}{2R}\right)\sum_{h=0}^{H-1}\left[\left(b_h + g_h x_{t-1}\right) n_{h,t}\right]\right\} \tag{7}$$

In the two type case of fundamentalists ($g_0 = 0$,$b_0 = 0$, $C_0 = C > 0$) and chartists

$(g_1 = g > 0, b_1 = 0, C_1 = 0)$ eq. (7) becomes

$$x_t = \frac{1}{DR\omega + 1}\left[\frac{1}{2} + g\left(D\omega + \frac{1}{2R}\right)n_{1,t}\right]x_{t-1} \tag{8}$$

In this case $n_{1,t}$, after replacing z_{ht} with (8) reads as follows:

$$n_{1,t} = \frac{1}{e^{-\beta[Dg(Rx_{t-2}-x_{t-1})x_{t-3}-C]} + 1} \tag{9}$$

It is convenient to introduce $m_t \equiv n_{0,t} - n_{1,t}$. Then eq. (8) becomes

$$x_t = \frac{1}{DR\omega + 1}\left[\frac{1}{2} - g\left(D\omega + \frac{1}{2R}\right)\left(\frac{m_{t-1} - 1}{2}\right)\right]x_{t-1} \tag{10}$$

where

$$m_{t-1} = \tanh\left(\frac{\beta}{2}\left[Dg_c\left(Rx_{t-2} - x_{t-1}\right)x_{t-3} - C\right]\right) \tag{11}$$

The market clearing case of BH98 is obtained for $\omega \to \infty$:

$$x_t = -\frac{g}{2R}\left(m_{t-1} - 1\right)x_{t-1} \tag{12}$$

The fundamental steady state solution $x = 0$ becomes unstable due to a pitchfork bifurcation which occurs when the following condition holds

$$g = R\left(1 + e^{-C\beta}\right) \tag{13}$$

In particular, the following lemma holds:

Lemma 1. *(Existence and stability of steady states). Let* $\overline{m}_f = \tanh\left(-\frac{\beta C}{2}\right)$, $\overline{m}_{nf} = 1 - \frac{2R}{g}$ *and* x^* *be the positive solution (if it exists) of* $\tanh\left[\frac{\beta}{2}\left(Dg\left(R-1\right)x^2 - C\right)\right] = \overline{m}_{nf}$. *Then:*

1. *For* $0 < g < R$ *the fundamental equilibrium* $E_0 = (0, \overline{m}_f)$ *is the unique, globally stable steady state*

2. *For $g \geq 2R$ the fundamental equilibrium is locally unstable and two other steady states exists: $E_1 = (x^*, \overline{m}_{nf})$ and $E_2 = (-x^*, \overline{m}_{nf})$*

3. *For $R < g < 2R$ there exists $\beta^* = \frac{1}{C} \log\left(\frac{1}{g/R-1}\right) \in (0, \infty)$ such that*

   (a) *if $\beta < \beta^*$ the fundamental equilibrium $E_0 = (0, \overline{m}_f)$ is the unique, globally stable steady state*

   (b) *if $\beta > \beta^*$ the fundamental equilibrium is locally unstable and two other steady states exists: $E_1 = (x^*, \overline{m}_{nf})$ and $E_2 = (-x^*, \overline{m}_{nf})$*

   (c) *if $\beta = \beta^*$ the fundamental and non fundamental equilibria coincide*

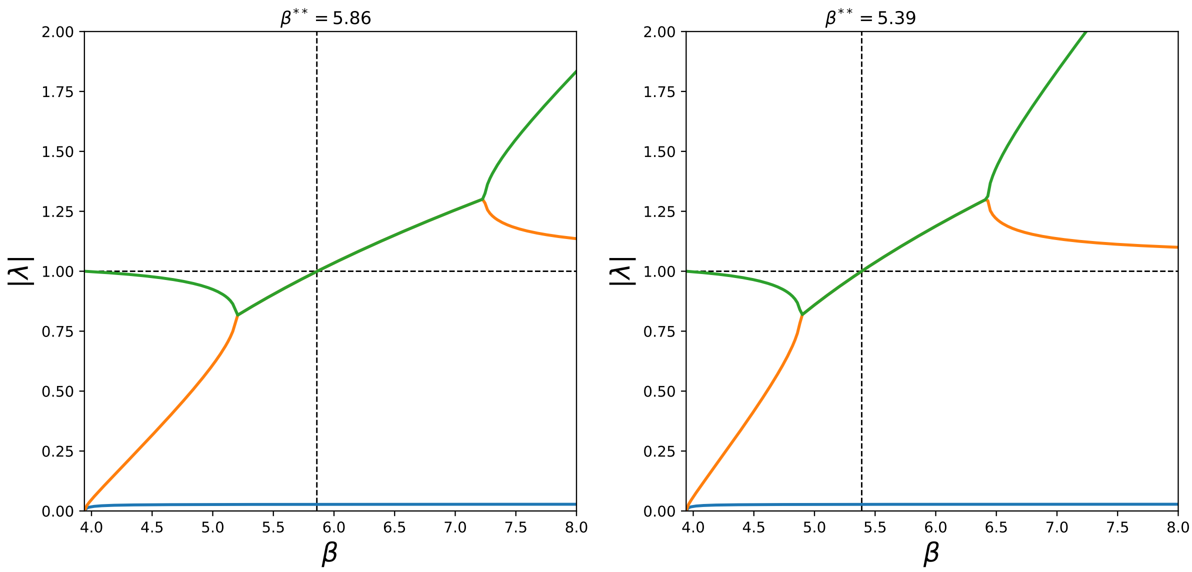
With Lemma 1 we recover the result of BH98, Lemma 2, which is obtained under the assumption of market clearing. Thus we see that the institutional framework of the market has no effect on the existence of non fundamental equilibria and on the stability of the fundamental equilibrium. In particular, the existence and stability of the steady states in BH98 does not depend on the assumption of market clearing. Moreover, the value of β^* is independent from the market setting.

The equivalent of lemma 3 in BH98 shows that there exists a critical value β^{**} above which the two non fundamental steady states become themselves unstable:

Lemma 2. *(Secondary Bifurcation). Let E_1 and E_2 be the non fundamental steady states as in Lemma 1. Assume $R < g < 2R$ and $C > 0$ and let β^* be the pitchfork bifurcation value. Further suppose that $R \in \left(1, \frac{4}{3}\right]$. Then there exists β^{**} such that E_1 and E_2 are stable for $\beta^* < \beta < \beta^{**}$ and unstable for $\beta > \beta^{**}$. For $\beta = \beta^{**}$ E_1 and E_2 exhibit a Hopf bifurcation.*

The numerical analysis of the value of the discriminant of the characteristic equation of the system at the non fundamental steady states and the numerical computation of its solutions show that there is an interval of values of β in which the discriminant is negative and thus there exists two conjugate complex roots which cross the unit circle at a value β^{**} (Fig. 2). From the bifurcation plot (Fig. 3) we see that at β^{**} the non fundamental steady state becomes unstable and that for higher values of β we observe an oscillating behavior between the fundamental and non fundamental steady states. The resulting dynamics is qualitatively identical to the 3-D system in BH98.

Differently from the 2-D system of Hommes *et al.* (2005), which considers the hypothesis of market making under a different framework, the critical value β^* in the present model remains the same of BH98. On the other hand, we see from Fig. 2 that β^{**} is decreasing in ω. In other terms, the likelihood that the market will settle at the non fundamental price is lower in the market clearing case ($\omega \to \infty$). Simmetrically, the more the market maker is inventory neutral ($\omega \to 0$), the more the market is likely to settle at the non fundamental steady state.

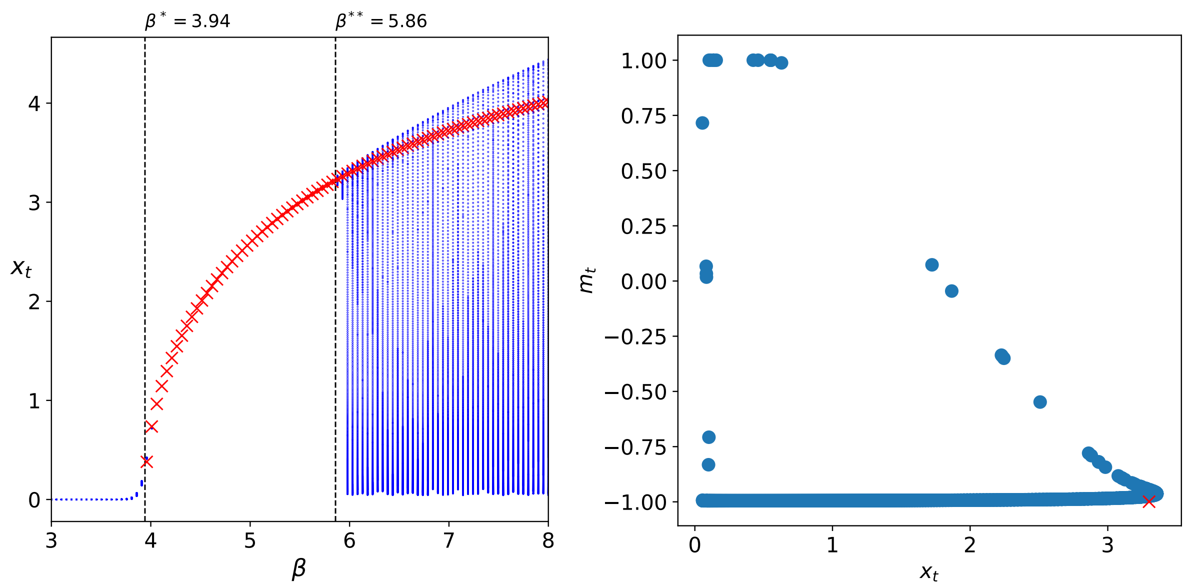


(a) $\omega = 1$ (b) $\omega \to \infty$

Figure 2: Solutions of the characteristic equation with $D = C = 1$, $R = 1.03$, $g = 1.05$ and $\beta^* = 3.94 \leq \beta \leq 8$
.

BH98 prove for $\beta \to \infty$ that, if $g > R^2$, the dynamic system is unstable (lemma 4). In our case it's possible to prove the following:

Lemma 3. *Assume $C > 0$ and $\beta \to \infty$. For $g > R$ the fundamental steady state*



(a) (b)

Figure 3: (a) Bifurcation diagram with $3 \leq \beta \leq 8$ and (b) periodic orbit for $\beta = 6$. The value of the other parameters are fixed as in Fig. 2, panel (a). The red crosses stand for the value of the non fundamental solution x^* obtained from eq. (24)

$E_0 = (0, -1)$ is locally unstable, with eigenvalues 0 and $\frac{R+(2DR\omega+1)g}{2R(DR\omega+1)}$. Let's fix $\bar{g}_0 = \frac{R\left(2DR^2\omega+2R-1\right)}{2DR\omega+1}$. There are two possibilities for the unstable manifold $W^u(E_0)$:

1. *if $g > \bar{g}_0$ then $W^u(E_0)$ equals the unstable eigenvector*

2. *if $R < g < \bar{g}_0$ then $W^u(E_0)$ is bounded and all orbits converge to E_0*

We see from Fig. 4 that the threshold $\bar{g}_0$ is decreasing in ω. This contrasts with the previous result: if the existence of periodic or quasi-periodic orbits does not depend on the institutional arrangement of the market, their stability depends on the behavior of the market maker. We recover the result of BH98 considering the limit $\omega \to \infty$, where $\bar{g}_0 = R^2$. Instead for $\omega \to 0$ we obtain the upper bound $\bar{g}_0 = R(2R-1)$. Thus the activity of the market maker, which absorbs the imbalance of supply and demand, makes the market less likely to suffer from severe instability which might call for an external intervention. Needless to say, this external intervention might be nevertheless required in order to support the market maker in her role under circumstances of severe market stress.

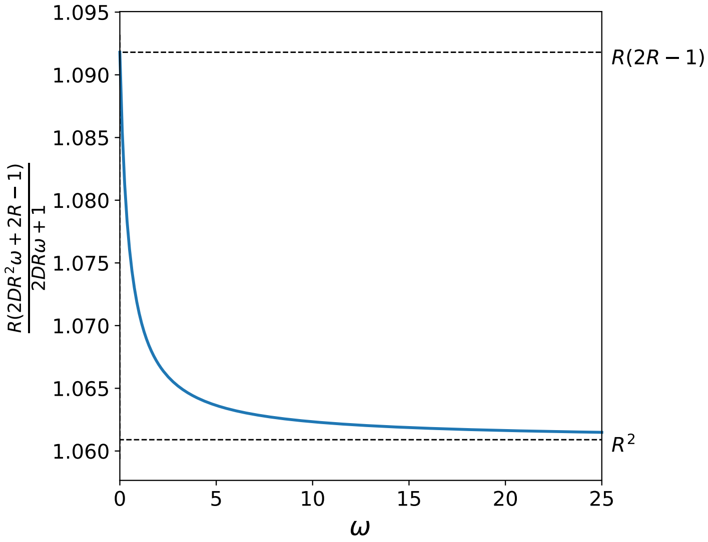


Figure 4: $\overline{g}_0$ for different values of ω. The values of the other parameters are: $D = 1$, $R = 1.03$.

4 Model with an "activist" market maker

We extend the previous model supposing that the market maker acts as a mean-variance optimizing speculator too. Her objective becomes

$$\max_{x_t, y_t} \left\{ (x_t - x_{t-1}) \sum_{h=0}^{H-1} n_{h,t} z^*_{h,t} - \omega \left(\sum_{h=0}^{H-1} n_{h,t} z^*_{h,t} \right)^2 + (g_d - R)\, x_t y_t - \frac{y_t^2}{2D} \right\} \tag{14}$$

where y_t represents the speculative demand of the market maker at t and g_d is her extrapolation parameter for the future price deviation, i.e. $E_d[x_{t+1}] = g_d x_t$. Substituting (6) in (14) and deriving for x_t and y_t we obtain the FOCs. In particular from the FOC with respect to y_t we obtain the optimal trading strategy:

$$y_t = D\,(g_d - R)\, x_t \tag{15}$$

Thus the market maker will trade in the same direction of x_t if the extrapolative component of her expectation is strong enough. Substituting (15) in the other FOC and solving for x_t we obtain the following price equation:

$$x_t = \frac{Rx_{t-1} + (2DR\omega + 1)\sum_{h=0}^{H-1}(b_h + g_h x_{t-1})\, n_{h,t}}{2R\,(DR\omega + 1) - (R - g_d)^2} \tag{16}$$

In the two type case of fundamentalists ($g_0 = 0$,$b_0 = 0$, $C_0 = C > 0$) and chartists ($g_1 = g_c > 0$,$b_1 = 0$,$C_1 = 0$) eq. (16) becomes

$$x_t = \frac{R + g_c\,(2DR\omega + 1)\ \ n_{1,t}}{2R\,(DR\omega + 1) - (R - g_d)^2} x_{t-1} \tag{17}$$

where $n_{1,t}$ is still given by eq. (9). After having replaced $n_{1,t} = \frac{1-m_t}{2}$ we obtain

$$x_t = \frac{R - g_c\,(2DR\omega + 1)\,\dfrac{m_{t-1} - 1}{2}}{2R\,(DR\omega + 1) - (R - g_d)^2} x_{t-1} \tag{18}$$

where m_{t-1} is still given by eq. (11). The BH98 equation (12) is obtained for $\omega \to \infty$ and $g_d \to R$, i.e. when the market maker doesn't accumulate inventories and doesn't trade (see eq. (15)). The fundamental steady state solution $x = 0$ becomes unstable due to a pitchfork bifurcation which occurs when the following condition holds

$$g_c = \left(1 + e^{-\beta C}\right)\left(R - \frac{(g_d - R)^2}{2DR\omega + 1}\right) \tag{19}$$

In particular, the following lemma holds.

Lemma 4. *(Existence and stability of steady states). Let*

$$\overline{m}_{nf} = 1 - \frac{2R}{g_c} + \frac{2\,(R - g_d)^2}{g_c\,(2DR\omega + 1)}$$

and x^* *be the positive solution (if it exists) of* $\tanh\left[\frac{\beta}{2}\left(Dg\,(R-1)\,x^2 - C\right)\right] = \overline{m}_{nf}$*. Further suppose that* $g_d \in \left(R - \sqrt{R(2DR\omega + 1)},\quad R + \sqrt{R(2DR\omega + 1)}\right)$*. Then:*

1. *For* $0 < g_c < R - \frac{(R-g_d)^2}{(2DR\omega+1)}$ *the fundamental equilibrium* $E_0 = (0, \overline{m}_f)$ *is the unique, globally stable steady state*

2. *For* $g_c > 2\left[R - \frac{(R-g_d)^2}{(2DR\omega+1)}\right]$ *the fundamental equilibrium is locally unstable and two other steady states exist:* $E_1 = (x^*, \overline{m}_{nf})$ *and* $E_2 = (-x^*, \overline{m}_{nf})$

3. *For* $0 \leq R - \frac{(R-g_d)^2}{(2DR\omega+1)} < g_c < 2\left[R - \frac{(R-g_d)^2}{(2DR\omega+1)}\right]$ *there exists* $\beta^* = \frac{1}{C}\log\left(\frac{1-\overline{m}_{nf}}{1+\overline{m}_{nf}}\right) \geq 0$ *such that*

   (a) *if* $\beta < \beta^*$ *the fundamental equilibrium* $E_0 = (0, \overline{m}_f)$ *is the unique, globally stable steady state*

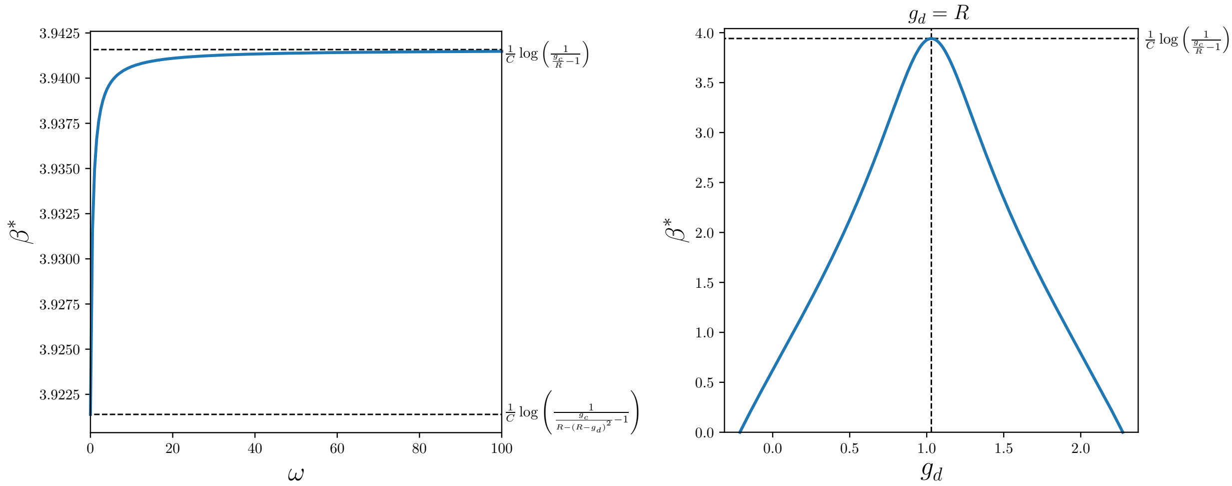
   (b) *if* $\beta > \beta^*$ *the fundamental equilibrium is locally unstable and two other steady states exist:* $E_1 = (x^*, \overline{m}_{nf})$ *and* $E_2 = (-x^*, \overline{m}_{nf})$

   (c) *if* $\beta = \beta^*$ *the fundamental and non fundamental equilibria coincide*

We see that β^* is increasing in ω and converges to $\beta^* = \frac{1}{C}\log\left(\frac{1}{g_c/R-1}\right)$ for $\omega \to \infty$ (see fig. 5, panel (a)). In this limit we recover the result of BH98 and of Sec. 3. This means that, the more the market maker is oriented against holding inventories, the more she can compensate for her own trading activity and try to keep the market anchored to the fundamental equilibrium. Except that in the limit $\omega \to \infty$, an activist market maker is more likely to make the market evolve towards a non fundamental steady state than a passive market maker or a Walrasian auctioneer. In particular, in the limit $\omega \to 0$ we obtain a nonnegative lower bound for β^*:

$$\lim_{\omega \to 0} \beta^* = \frac{1}{C}\log\left(\frac{1}{\frac{g_c}{R-(R-g_d)^2} - 1}\right) \tag{20}$$

From fig. 5 we see also that β^* is increasing in g_d up to $g_d = R$ and decreasing for $g_d > R$. Thus the effect of an increasingly extrapolative chartist market maker on market stability is nonlinear: stabilizing for $g_d < R$ and destabilizing for $g_d > R$. When $g_d = R$, we recover again the same β^* of BH98 and of Sec. 3. For any other $g_d \neq R$ we have instead that β^* is lower than in those two models. Thus an activist market maker makes the market less stable than a passive market maker except when $g_d = R$. In this case we recover the same level of stability of Sec 3 indeed because the market maker is not trading at all (see eq. (15)). Moreover, the effect of a chartist market maker is to improve the stability of the market with respect to a fundamentalist market maker ($g_d = 0$) for any $g_d \in (0, 2R)$. From eq. (15) we see that a fundamentalist market maker trades in the opposite direction of speculators and this might explain the negative effect on market stability.



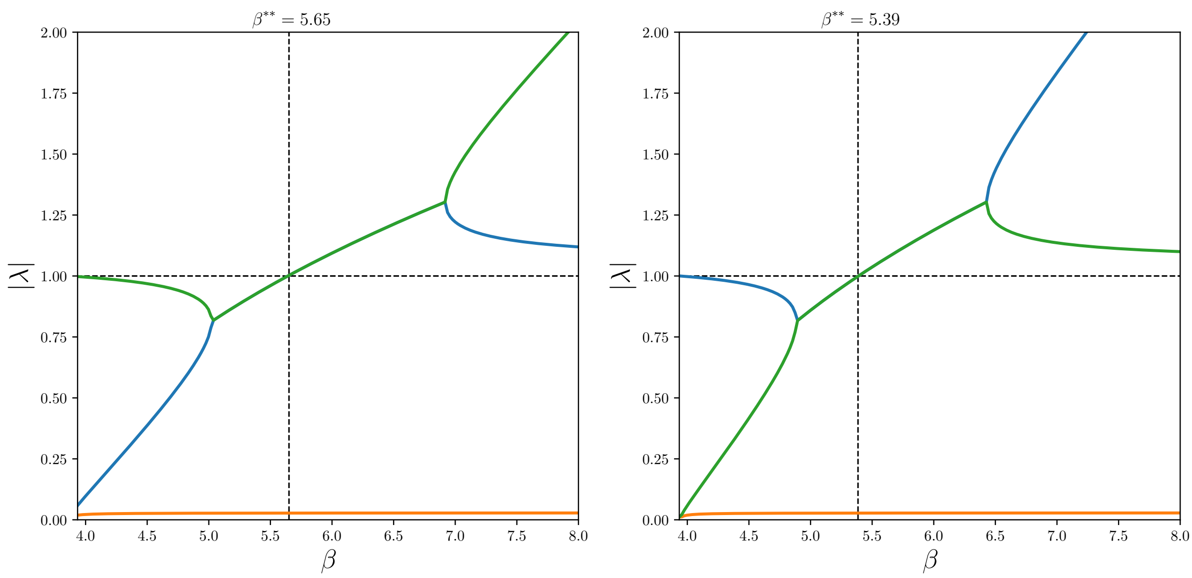
(a) $g_d = 1.05$

(b) $\omega = 1$

Figure 5: Value of β^* for different values of ω and g_d. The other parameters are fixed at the following values: $D = C = 1$, $R = 1.03$, $g_c = 1.05$.

Lemma 5. *(Secondary Bifurcation). Let E_1 and E_2 be the non fundamental steady states as in Lemma 4. Assume $0 \leq R - \frac{(R-g_d)^2}{(2DR\omega+1)} < g_c < 2\left[R - \frac{(R-g_d)^2}{(2DR\omega+1)}\right]$ and $C > 0$ and let $\beta^* \geq 0$ be the pitchfork bifurcation value. Further suppose that $R \leq \frac{4}{3}$ and $g_d \in \left(R - \sqrt{2R}, R + \sqrt{2R}\right)$. Then there exists β^{**} such that E_1 and E_2 are stable for $\beta^* < \beta < \beta^{**}$ and unstable for $\beta > \beta^{**}$. For $\beta = \beta^{**}$ E_1 and E_2 exhibit a Hopf bifurcation.*

The numerical analysis of the discriminant of the characteristic equation at the non fundamental steady states and the numerical computation of its solutions yield essentially the same results of the previous model. There is an interval of values of β in which the discriminant is negative and thus there exist two conjugate complex roots which cross the unit circle at a value β^{**}. At this critical value the non fundamental steady state becomes unstable and for higher values of β we observe an oscillating behavior between the fundamental and non fundamental steady states. The resulting dynamics is qualitatively identical to the 3-D system in BH98. We remark that, using the same parameter values $\omega = D = C = 1$ and $R = 1.03$, plus $g_d = 1.05$ we obtain that the critical values β^{**} are lower than in Sec. 3 (Fig. 2). Moreover, β^{**} is decreasing in ω and thus the same considerations apply, namely that market making is making the non fundamental steady states more likely to be unstable.



(a) $\omega = 1$ (b) $\omega \to \infty$

Figure 6: Solutions of the characteristic equation with $D = C = 1$, $R = 1.03$, $g_c = 1.05$, $g_d = 1.1$ and $\beta^* = 3.86 \leq \beta \leq 8$.
.

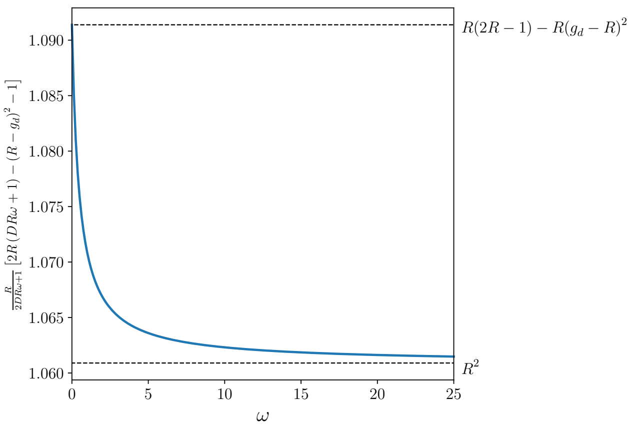
The equivalent of Lemma 3 of Sec. 3 is stated as follows:

Lemma 6. *Assume* $C > 0$, $g_d \in \left(R - \sqrt{R(2DR\omega + 1)}, \quad R + \sqrt{R(2DR\omega+1)}\right)$ *and* $\beta \to \infty$. *Suppose that* $g_c > R - \frac{(R-g_d)^2}{(2DR\omega+1)} \geq 0$ *so that the fundamental steady state* $E_0 = (0, -1)$ *is locally unstable. Let's fix* $\bar{g}_1 = \frac{R}{2DR\omega+1}\left[2R\left(DR\omega+1\right) - \left(R - g_d\right)^2 - 1\right]$.

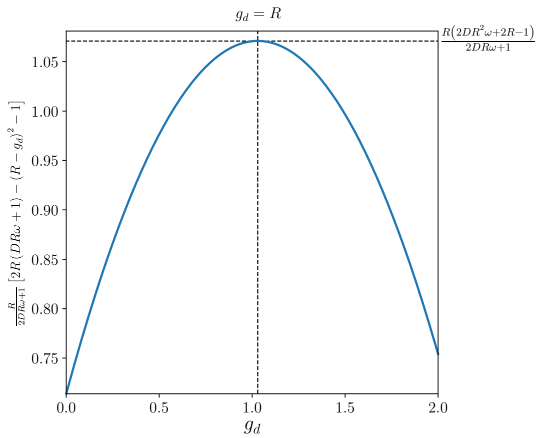
There are two possibilities for the unstable manifold $W^u(E_0)$:

1. *if $g_c > \bar{g}_1$ then $W^u(E_0)$ equals the unstable eigenvector*
2. *if $R - \frac{(R-g_d)^2}{(2DR\omega+1)} < g_c < \bar{g}_1$ then $W^u(E_0)$ is bounded and all orbits converge to E_0*

From Fig. 7 we see that the effect of ω is the same of the previous model, thus market making still has a stabilizing effect compared to market clearing. On the other hand we see from panel (b) that the speculative activity of the market maker makes the market more likely of being destabilized with a behavior that is equivalent to the one of fig. 5, panel (b).



(a)



(b)

Figure 7: Value of $\overline{g}_1$ for different values of ω and g_d. The other parameters are fixed at the following values: $D = C = 1$, $R = 1.03$, $g_d = 1.05$ (panel (a)), $\omega = 1$ (panel (b)).

5 Conclusions

The dynamic behavior of the models presented in this paper is qualitatively identical to the one described in BH98. This shows that the complex dynamics of that model is independent from the institutional framework of the market and in particular from the assumption of market clearing. While this result was already established, under a simpler setting, by Hommes *et al.* (2005), I show also that the market maker has conflicting effects on the stability of the market. She act as a stabilizer when she allows for market imbalances, and as a destabilizer when she manages aggressively her inventories or when she trades actively. The more stable institutional framework is one in which the market maker is inventory neutral and doesn't trade actively. Even in this scenario the typical complex behavior of BH98 occurs.

In this paper the assumption of market clearing is replaced by the assumption that markets are always liquid for the market maker. This hypothesis is critical to achieve positive profits since it allows the market maker to adjust her inventory at a favourable price. By making this assumption, we are implicitly introducing some liquidity provider of last resort in the model, who might not necessarily be a speculator. Who might play this role in the FX market? One option is provided by hedgers, like non financial corporations, who represent important actors on the FX market. The model of Evans and Lyons (2002) include two distinct classes of agents: the first one, represented by speculators, demands liquidity from FX dealers at the beginning of the day, the second one, represented by hedgers or "profit takers", supplies to dealers the necessary liquidity to balance the market. Empirical evidence confirms that financial customers demand liquidity while hedgers are net liquidity providers (King *et al.*, 2013). Alternatively, we should not forget banks: given the pivotal role of the banking sector in the FX market, their highly elastic supply of liquidity accomodates the needs of the operators on the money markets for the domestic currency and, through liquidity swaps arranged by central banks, for the most traded foreign currencies. A more careful assessment of these hypotheses is left for future research.

I thank Mikhail Anufriev and the participants to the 30$^{\text{th}}$ EAEPE Conference in Nice, to the GENED 6$^{\text{th}}$ annual meeting in Kiel and to the Department Seminar at DISEI for their useful comments. All the usual disclaimers apply.

A Proofs

Proof of Lemma 1. Starting from eq. (10), the steady state must satisfy the following:

$$x^* = \frac{1}{DR\omega + 1}\left[\frac{1}{2} - g\left(D\omega + \frac{1}{2R}\right) g\,\frac{m^* - 1}{2}\right] x^* \tag{21}$$

where

$$m^* = n_0^* - n_1^* = \tanh\left[\frac{\beta}{2}\left(-C + Dg\left(R-1\right)\left(x^*\right)^2\right)\right] \tag{22}$$

The non fundamental solution is obtained by solving the following equation

$$m^* = 1 - \frac{2R}{g} \tag{23}$$

We obtain that

$$x^* = \sqrt{\frac{C\beta + \log\left(\frac{g}{R} - 1\right)}{D\beta g\left(R-1\right)}} \tag{24}$$

Since x^* must be real, the following must hold:

$$\left(\frac{g}{R} - 1\right) e^{C\beta} \geq 1 \tag{25}$$

solving for g we obtain $g \geq R\left(1 + e^{-C\beta}\right)$. Letting $\beta \to \infty$ we obtain the first claim, letting $\beta \to 0$ the second claim regarding the existence of two symmetric non fundamental steady states.

The positive eigenvalue at the fundamental steady state is

$$\lambda = \frac{2R + g\left(2DR\omega + 1\right)\left[\tanh\left(\frac{\beta C}{2}\right) + 1\right]}{4R\left(DR\omega + 1\right)} \tag{26}$$

Solving for g the inequality $\lambda < 1$ we obtain that the fundamental steady state is locally stable if $0 < g < R\left(1 + e^{-C\beta}\right)$. Letting $\beta \to \infty$ we obtain the first claim, letting $\beta \to 0$ the second claim regarding the stability of the fundamental steady state.

Solving for β the equality $\lambda = 1$ we obtain the critical value

$$\beta^* = \frac{1}{C} \log \left(\frac{1}{g/R - 1} \right) \tag{27}$$

We see that for $R < g < 2R$ we have that $\beta^* \in (0, \infty)$. The last claim is proved by substituting β^* into eq. (24) since in this case we obtain $x^* = 0$.

$\square$

Proof of Lemma 2. The characteristic equation for the stability of the non fundamental steady states is

$$P(\lambda, K) = \lambda^3 - \lambda^2 \left(\frac{K}{R} + 1 \right) + K\lambda + K \left(1 - \frac{1}{R} \right) = 0 \tag{28}$$

where

$$K = -\frac{g_c C}{8\,(R-1)\,(DR\omega + 1)} \,(\beta - \beta^*)\,(2DR\omega + 1)\left(\overline{m}_{nf}^2 - 1\right) \tag{29}$$

$\overline{m}_{nf} \equiv 1 - \frac{2R}{g}$ and β^* is defined as in eq. (27). Under the hypotheses we have $\beta > \beta^*$ and $-1 < \overline{m}_{nf} < 0$, thus $K > 0$.

The critical points of $P(\lambda, K)$ are

$$x_{critical} = \frac{1}{3R} \left(K + R \pm \sqrt{(K+R)^2 - 3KR^2} \right) \tag{30}$$

We see that for $K > 0$ and $(K+R)^2 - 3KR^2 \geq 0$, the two critical points are real and positive. Since the derivative of $G(K) \equiv (K+R)^2 - 3KR^2$ is increasing in K, while the second derivative is strictly positive, $G(K)$ has a global minimum at $K^* = \frac{R}{2}(3R-2) > 0$. If we substitute back K^* into G we obtain that the minimum of G is $G(K^*) = -\frac{9R^4}{4} + 3R^3$ which is nonnegative if the following holds

$$1 < R \leq \frac{4}{3} \tag{31}$$

which we assume is true from now on. From the foregoing we derive

$$\begin{aligned}P(-1,K) &= -\frac{2}{R}(K+R) < 0\\ P(0,K) &= \frac{K}{R}(R-1) > 0\\ P(1,K) &= \frac{2K}{R}(R-1) > 0\end{aligned}$$

Thus the smallest of the three roots is always real and comprised between -1 and 0. The two other roots instead have always a positive real part. In fact the positive larger critical point lies to the left of the largest root if the latter is real, or otherwise it coincides with the real part of the two complex conjugate roots. On the other hand, the positive smaller critical point lies to the left of the smaller of the two real roots or to the left of the real part of the two complex conjugate roots.

In order to show that the largest root crosses the unit circle for $\beta \to \infty$ it suffices to show that the largest critical point is increasing in K, since we know that K is increasing in β and since the largest critical point is a lower bound for the absolute value of the largest root in absolute terms. Now we differentiate the largest critical point wrt K:

$$\frac{\partial x_{critical}}{\partial K} = \frac{1}{3R}\left(1 + \frac{K - \frac{3R^2}{2} + R}{\sqrt{-3KR^2 + (K+R)^2}}\right) \tag{32}$$

We consider the following lower bound:

$$\frac{1}{3R}\left(1 + \frac{-\frac{3R^2}{2} + R}{K+R}\right)$$

which is increasing in K for $R \geq 1$. We need to prove that it is nonnegative. Solving the inequality for K we obtain

$$K \geq \frac{R}{2}(3R-4) \tag{33}$$

which is automatically satisfied since $K > 0$ as soon as $R < 4/3$.

In order to prove that we have a Hopf bifurcation we observe that if the largest real root would become equal to unity while K increases, we would necessarily have that $P(1, K) = 0$ for some value of K, something which contradicts our previous statements.

$\square$

Proof of Lemma 3. Following BH98, in order to prove the claim we need to show that, when the fundamental steady state is unstable, the system returns to the fundamental steady state for some $T > 0$ if and only if $g < \frac{R\left(2DR^2\omega+2R-1\right)}{2DR\omega+1}$.

When $\beta \to \infty$ we have that $\overline{m}_f \equiv \tanh\left(-\beta\frac{C}{2}\right) \to -1$ and the fundamental steady state is $E_0 = (0, -1)$. The eigenvalues at the fundamental steady state are $(0, 0, \lambda_\infty)$ with

$$\lambda_\infty = \frac{R + (2DR\omega + 1)g}{2R\,(DR\omega + 1)} \tag{34}$$

We suppose that $g > R$, thus the fundamental steady state is unstable since $\lambda_\infty > 1$. The eigenvector associated with λ_∞ is

$$\begin{bmatrix} \frac{(2DRg\omega+R+g)^2}{4R^2(DR\omega+1)^2} \\ \frac{2DRg\omega+R+g}{2R(DR\omega+1)} \\ 1 \end{bmatrix} \tag{35}$$

We know that the system evolves according to eqs. (10) and (11), which we reproduce here for convenience of the reader:

$$x_t = \frac{2R - g\,(2DR\omega + 1)\,(m_{t-1} - 1)}{4R\,(DR\omega + 1)} x_{t-1} \tag{10}$$

$$m_{t-1} = \tanh\left(\frac{\beta}{2}\left(Dg_c\left(Rx_{t-2} - x_{t-1}\right)x_{t-3} - C\right)\right) \tag{11}$$

Let's consider the following expression

$$C_t \equiv Dg\left(Rx_{t-1} - x_t\right)x_{t-2} \tag{36}$$

we see that for $\beta \to \infty$

$$m_t = \begin{cases} +1 & \text{if} \quad C_t > C \\ -1 & \text{otherwhise} \end{cases} \tag{37}$$

Let's suppose that, starting from the fundamental steady state, a small shock occurs at $t = -2$ and is propagated until $t = 0$. Then we have

$$\begin{aligned} x_{-2} &= \epsilon \\ x_{-1} &= \frac{2R - g\left(2DR\omega + 1\right)\left(m_{-2} - 1\right)}{4R\left(DR\omega + 1\right)}\epsilon \\ x_0 &= \left[\frac{2R - g\left(2DR\omega + 1\right)\left(m_{-1} - 1\right)}{4R\left(DR\omega + 1\right)}\right]\left[\frac{2R - g\left(2DR\omega + 1\right)\left(m_{-2} - 1\right)}{4R\left(DR\omega + 1\right)}\right]\epsilon \end{aligned}$$

By hypothesis we know that $m_{-3} = -1$. Furthermore $C_{-2} = C_{-1} = 0$ thus $m_{-2} = m_{-1} = -1$. As a consequence we obtain the following:

$$\begin{aligned} x_{-1} &= \lambda_\infty \epsilon \\ x_0 &= \lambda_\infty^2 \epsilon \end{aligned}$$

From our hypotheses we see that the system is on an explosive path. Any trajectory starting in a neighborhood of the fundamental steady state will move along the unstable eigenvector until $m_t = -1$ and thus will diverge to infinity unless $m_T = 1$ for some $T > 0$. In fact in this case we have that

$$x_{T+1} = \frac{x_T}{2\left(DR\omega + 1\right)} \tag{38}$$

and thus $x_{T+t} \to 0$ for $t \to \infty$ as long as $C_{T+t} > C$. This eventuality depends on the evolution of the value of C_t over time. In particular, if $C_t \to \infty$ for $t \to \infty$ then for some $T > 0$ the conclusion follows.

Let's suppose that $C_{t-k} < C$ for all $0 \leq k \leq t$ (otherwise the conclusion already follows) so that $m_{t-k} = -1$. Substituting in C_t the iterated values obtained from an initial shock ϵ occurred at $t = -2$ we obtain

$$C_t = \lambda_\infty^{2t} D\epsilon^2 g \frac{g\left(2DR\omega + 1\right) + R}{4R^2\left(DR\omega + 1\right)^2}\left[2DR\omega(R^2 - g) + R(2R - 1) - g\right] \tag{39}$$

which will diverge to $\pm\infty$ depending on the sign of the rightmost term. Thus the conclusion follows from the following condition

$$g < \frac{R\left(2DR^2\omega + 2R - 1\right)}{2DR\omega + 1} \tag{40}$$

We recover the result of BH98 considering the limit $\omega \to \infty$ where we obtain that $g < R^2$

.

□

Proof of Lemma 4. The steady state solutions must satisfy the following equation:

$$x^* = \frac{R - \left(2DR\omega + 1\right) g_c \dfrac{m^* - 1}{2}}{2R\left(DR\omega + 1\right) - \left(R - g_d\right)^2} x^* \tag{41}$$

where m^* is defined as in eq. (22). The non fundamental solution is obtained by solving the following equation:

$$m^* = \overline{m}_{nf} \tag{42}$$

where

$$\overline{m}_{nf} \equiv 1 - \frac{2R}{g_c} + \frac{2\left(R - g_d\right)^2}{g_c\left(2DR\omega + 1\right)} \tag{43}$$

We obtain that

$$x^* = \sqrt{\frac{C\beta + \log\left(\frac{1+\overline{m}_{nf}}{1-\overline{m}_{nf}}\right)}{D\beta g_c\left(R - 1\right)}} \tag{44}$$

Since x^* must be real, the following must hold:

$$e^{\beta C}\frac{1 + \overline{m}_{nf}}{1 - \overline{m}_{nf}} \geq 1 \tag{45}$$

Solving this condition for g_c we obtain

$$g_c \geq \left(R - \frac{(R-g_d)^2}{2DR\omega+1} \right) \left(1+e^{-\beta C}\right) \tag{46}$$

Letting $\beta \to \infty$ we obtain the first claim, letting $\beta \to 0$ the second claim regarding the existence of two symmetric non fundamental steady states.

The non trivial eigenvalue at the fundamental steady state is

$$\lambda = \frac{2R + g_c\,(2DR\omega+1)\left[\tanh\left(\frac{\beta C}{2}\right)+1\right]}{4R\,(DR\omega+1) - 2\,(R-g_d)^2} \tag{47}$$

The reader can easily check that λ is positive and finite under the hypotheses. We see that λ is increasing in g_c. Solving the inequality $\lambda < 1$ for g_c we obtain that the fundamental steady state is locally stable if the following condition holds

$$0 < g_c < \left(R - \frac{(R-g_d)^2}{2DR\omega+1} \right) \left(1+e^{-\beta C}\right) \tag{48}$$

Letting $\beta \to \infty$ we obtain the first claim, letting $\beta \to 0$ the second claim regarding the stability of the fundamental steady state.

Solving the equality $\lambda = 1$ for β we obtain the critical value β^*:

$$\beta^* = \frac{1}{C} \log\left(\frac{1-\overline{m}_{nf}}{1+\overline{m}_{nf}} \right) \tag{49}$$

It's easy to check that for $0 \leq R - \frac{(R-g_d)^2}{(2DR\omega+1)} < g_c < 2\left[R - \frac{(R-g_d)^2}{(2DR\omega+1)}\right]$ we have that $\beta^* \in (0,\infty)$. The last claim of the lemma is proved by substituting β^* into eq. (44) since in this case we obtain $x^* = 0$.

□

Proof of Lemma 5. The characteristic equation for the stability of the non fundamental steady states is

$$P(\lambda, K) = \lambda^3 - \lambda^2 \left(\frac{K}{R} + 1 \right) + K\lambda + K \left(1 - \frac{1}{R} \right) = 0 \tag{50}$$

where

$$K = -\frac{R g_c C}{(R-1)\left[8R\left(DR\omega+1\right) - 4\left(R-g_d\right)^2\right]} \left(\beta - \beta^*\right)\left(2DR\omega + 1\right)\left(\overline{m}_{nf}^2 - 1\right) \tag{51}$$

and $\overline{m}_{nf}$ and β^* are defined as in eqs. (43) and (49) respectively. Under the hypotheses we have $\beta > \beta^*$ and $-1 < \overline{m}_{nf} < 0$, thus the sign of K depends on the sign of $F(K) \equiv 8R\left(DR\omega+1\right) - 4\left(R-g_d\right)^2$ which is increasing in ω. We have that $F(K) > 0$ if the following condition holds:

$$\omega > \frac{1}{DR^2}\left(\frac{1}{2}\left(R-g_d\right)^2 - R\right) \tag{52}$$

The expression in parenthesis on the RHS is negative for $g_d \in \left(R - \sqrt{2R}, R + \sqrt{2R}\right)$, which we assume is true. Thus the condition (52) is satisfied by our assumptions on ω and we obtain that $K > 0$. Following the same arguments of Lemma 2, we obtain the conclusion.

□

Proof of Lemma 6. The argument follows the same lines of the proof of Lemma 2. In particular, the eigenvalues at the fundamental steady state for $\beta \to \infty$ are $(0, 0, \lambda_\infty)$ with

$$\lambda_\infty = \frac{(2DR\omega + 1)g_c + R}{2R(DR\omega + 1) - (R - g_d)^2} \tag{53}$$

The reader can check that, under the hypotheses, λ_∞ is finite and greater than unity. The system evolves according to eq. (18) which we reproduce for convenience of the reader:

$$x_t = \frac{R - g_c\left(2DR\omega + 1\right)\dfrac{m_{t-1} - 1}{2}}{2R\left(DR\omega + 1\right) - \left(R - g_d\right)^2} x_{t-1} \tag{18}$$

where m_{t-1} is given by eq. (11). Introducing a shock at $t = -2$ and repeating the steps of lemma 3 we obtain that $x_0 = \lambda_\infty^2 \epsilon$ and $m_0 = -1$. As long as $m_{t-1} = -1$ for $t \geq 2$, we obtain that $x_t = \lambda_\infty^{2+t} \epsilon$ and the system is on an explosive path. Instead if $m_T = 1$ for some $T > 0$ we have that

$$x_{T+1} = \frac{R}{2R\,(DR\omega + 1) - (R - g_d)^2} x_T \tag{54}$$

The reader can check that under the hypotheses made on g_d the coefficient on the RHS is positive and smaller than unity, then $x_{T+t} \to 0$ for $t \to \infty$ as long as $C_{T+t} > C$. The expression for C_t in this case becomes

$$C_t = \lambda_\infty^{2t} D \epsilon^2 g_c \frac{g_c\,(2DR\omega + 1) + R}{\left[2R\,(DR\omega + 1) - (g_d - R)^2\right]^2} \\ \Big(2DR\omega(R^2 - g_c) + R(2R - 1) - R(R - g_d)^2 - g_c\Big) \tag{55}$$

which diverges to $\pm\infty$ depending on the sign of the rightmost term. Thus the conclusion follows from the following condition:

$$g_c < \frac{R}{2DR\omega + 1} \left[2R\,(DR\omega + 1) - (R - g_d)^2 - 1\right] \tag{56}$$

We recover the result of BH98 considering the limit $\omega \to \infty$ where we obtain $g_c < R^2$. In the limit $\omega \to 0$ we obtain instead the following:

$$g_c < R\left[2R - 1 - (R - g_d)^2\right] \tag{57}$$

$\square$

References

Mikhail Anufriev, Valentyn Panchenko,Asset prices, traders' behavior and market design,Journal of Economic Dynamics and Control,Volume 33, Issue 5,2009,Pages 1073-1090.

William A. Brock, Cars H. Hommes,Heterogeneous beliefs and routes to chaos in a simple asset pricing model, Journal of Economic Dynamics and Control, Volume 22, Issues 8–9, 1998, Pages 1235-1274.

Geir Høidal Bjønnes, Dagfinn Rime,Dealer behavior and trading systems in foreign exchange markets, Journal of Financial Economics,Volume 75, Issue 3,2005,Pages 571-605.

Jun Cai, Yan-Leung Cheung, Raymond S.K. Lee, Michael Melvin,'Once-in-a-generation' yen volatility in 1998: fundamentals, intervention, and order flow,Journal of International Money and Finance,Volume 20, Issue 3,2001,Pages 327-347.

Alessandro Carraro, Giorgio Ricchiuti,Heterogeneous fundamentalists and market maker inventories,Chaos, Solitons & Fractals,Volume 79,2015,Pages 73-82.

Yin-Wong Cheung, Menzie David Chinn, Currency traders and exchange rate dynamics: a survey of the US market, Journal of International Money and Finance, Volume 20, Issue 4, 2001, Pages 439-471

Richard H. Day, Weihong Huang,Bulls, bears and market sheep,Journal of Economic Behavior & Organization,Volume 14, Issue 3,1990,Pages 299-329.

Evans, Martin D. D., and Richard K. Lyons. "Order Flow and Exchange Rate Dynamics." Journal of Political Economy, vol. 110, no. 1, 2002, pp. 170–180. JSTOR, JSTOR, www.jstor.org/stable/10.1086/324391.

Michael R. King, Carol L. Osler, Dagfinn Rime,The market microstructure approach to foreign exchange: Looking back and looking forward,Journal of International Money and Finance,Volume 38,2013,Pages 95-119,

Kyle, Albert S. “Continuous Auctions and Insider Trading.” Econometrica, vol. 53, no. 6, 1985, pp. 1315–1335., www.jstor.org/stable/1913210.

Lawrence R. Glosten, Paul R. Milgrom, Bid, ask and transaction prices in a specialist market with heterogeneously informed traders, Journal of Financial Economics, Volume 14, Issue 1, 1985, Pages 71-100.

Ho, T., Stoll, H.R., 1981. Optimal dealer pricing under transactions and return uncertainty. Journal of Financial Economics 9, 47–73.

Cars Hommes, Hai Huang, Duo Wang,A robust rational route to randomness in a simple asset pricing model,Journal of Economic Dynamics and Control,Volume 29, Issue 6,2005,Pages 1043-1072.

Huang, R.D., Stoll, H.R., 1997. The components of the bid–ask spread: A general approach. Review of Financial Studies 10, 995–1034.

Lyons, Richard K. (1997). A Simultaneous Trade Model of the Foreign Exchange Hot Potato. Journal of International Economics, 42, pp. 275–298.

Lux, Thomas. “Herd Behaviour, Bubbles and Crashes.” The Economic Journal, vol. 105, no. 431, 1995, pp. 881–896. JSTOR, JSTOR, www.jstor.org/stable/2235156.

Madhavan, A., Smidt, S., 1991. A Bayesian model of intraday specialist pricing. Journal of Financial Economics 30, 99–134.

Manaster, S., Mann, S.C., 1996. Life in the pits: competitive market making and inventory control. Review of Financial Studies 9, 953–976.

Osler, Carol L. and Vandrovych, Vitaliy, Hedge Funds and the Origins of Private Information in Currency Markets (April 2009). Available at SSRN: https://ssrn.com/abstract=1484711 or http://dx.doi.org/10.2139/ssrn.1484711.

Carol L. Osler, Alexander Mende, Lukas Menkhoff,Price discovery in currency markets,Journal of International Money and Finance,Volume 30, Issue 8,2011,Pages 1696-1718

Peiers, Bettina. “Informed Traders, Intervention, and Price Leadership: A Deeper View of the Microstructure of the Foreign Exchange Market.” The Journal of Finance, vol. 52, no. 4, 1997, pp. 1589–1614. JSTOR, JSTOR, www.jstor.org/stable/2329448.

Westerhoff, Frank, (2009), Exchange Rate Dynamics: A Nonlinear Survey, ch. 11 in , Handbook of Research on Complexity, Edward Elgar Publishing.

Frank Westerhoff, 2003. ”Market-maker, inventory control and foreign exchange dynamics,” Quantitative Finance, vol. 3(5), pages 363-369.

Mei Zhu, Carl Chiarella, Xue-Zhong He, Duo Wang,Does the market maker stabilize the market?,Physica A: Statistical Mechanics and its Applications,Volume 388, Issues 15–16,2009,Pages 3164-3180.