Answers to Odd-Numbered Exercises

CHAPTER 0

Section 0.1, page 9

1. $a = 4, \quad b = 2, \quad c = 9, \quad d = -1$

3. (a) Not possible (b) Not possible

(c) $\begin{bmatrix} 2 & 3 \\ 3 & 1 \\ 1 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 4 & 4 & 4 \\ 2 & 4 & 4 \\ 1 & 11 & 4 \end{bmatrix}$

5. $\mathbf{AB} = \begin{bmatrix} 9 & 11 \\ 10 & 2 \end{bmatrix}, \quad \mathbf{BA} = \begin{bmatrix} -5 & 1 \\ 12 & 16 \end{bmatrix}$

7. $\mathbf{AC} = \mathbf{BC} = \begin{bmatrix} -3 & -12 \\ -8 & -32 \end{bmatrix}$

9. $$\begin{aligned} 3x_1 - 2x_2 + 5x_3 + 4x_4 &= 1 \\ 4x_1 + 2x_2 + x_3 &= -3 \\ 3x_1 + 4x_2 - 2x_3 + x_4 &= 5 \end{aligned}$$

Section 0.2, page 20

1. $\begin{bmatrix} 1 & 0 & 0 & 11 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -4 \end{bmatrix}$ 3. $\begin{bmatrix} 1 & 0 & -\frac{7}{3} & 2 & -\frac{13}{3} \\ 0 & 1 & 2 & -1 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

5. (a) $x = 1, \quad y = 2, \quad z = -2$
   (b) No solution

7. (a) No solution
   (b) $x = 3, \quad y = -2, \quad z = 0, \quad w = 1$

9. (a) No solution
   (b) $x = \frac{7}{3} - \frac{7}{9}r, \quad y = \frac{4}{3} + \frac{8}{9}r, \quad z = r$, where r is any real number.

11. (a) $x = 0, \quad y = 0, \quad z = 0$
    (b) $x = -r, \quad y = 0, \quad z = 0, \quad w = r$, where r is any real number.

13. (i) $a = -3$
    (ii) a = any real number except 3 or -3
    (iii) $a = 3$

Section 0.3, page 27

1. $\begin{bmatrix} \frac{4}{5} & -\frac{1}{5} \\ -\frac{3}{5} & \frac{2}{5} \end{bmatrix}$

3. (a) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -3 & 1 \end{bmatrix}$

5. (a) $\begin{bmatrix} -\frac{5}{7} & \frac{3}{7} \\ \frac{4}{7} & -\frac{1}{7} \end{bmatrix}$ (b) No inverse (c) $\begin{bmatrix} \frac{14}{9} & -\frac{1}{3} & -\frac{1}{9} \\ -\frac{2}{3} & 0 & \frac{1}{3} \\ -\frac{5}{9} & \frac{1}{3} & \frac{1}{9} \end{bmatrix}$

7. (a) $\begin{bmatrix} \frac{2}{5} & -\frac{3}{10} \\ \frac{1}{5} & \frac{1}{10} \end{bmatrix}$ (b) $\begin{bmatrix} \frac{1}{2} & 0 & -\frac{1}{2} \\ \frac{7}{4} & -\frac{3}{2} & -\frac{5}{4} \\ -1 & 1 & 1 \end{bmatrix}$ (c) No inverse

9. (a) Does not exist
   (b) $\begin{bmatrix} 4 & -2 & -\frac{3}{2} \\ -\frac{13}{7} & 1 & \frac{9}{14} \\ -\frac{12}{7} & 1 & \frac{11}{14} \end{bmatrix}$
   (c) Does not exist

Section 0.4, page 32

3. (b) and (c)
5. (a) and (b)

Section 0.5, page 41

1. (a) and (b)
3. (a) and (b)
5. (c): $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} + \begin{bmatrix} 3 \\ 4 \\ -2 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \\ -1 \end{bmatrix}$

   (d): $\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} + 2\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 1 \end{bmatrix}$
7. (a)
9. (a)
11. (c): $\begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} - 2\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} + 3\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$
13. (a) $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix}$ (c) $\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$ (d) $\begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix}$
19. 2

CHAPTER 1

Section 1.1, page 60

1. Let x = amount of PEST (in kilograms) and
   y = amount of BUG (in kilograms).

   Minimize $z = 3x + 2.5y$
   subject to

$$30x + 40y \geq 120$$
$$40x + 20y \leq 80$$
$$x \geq 0, \quad y \geq 0.$$

   To change to standard form, change the objective function and the first constraint.
3. Let x = number of Palium pills prescribed per day and
   y = number of Timade pills prescribed per day.

Minimize $z = 0.4x + 0.3y$
subject to

$$\begin{aligned} 4x + 2y &\geq 10 \\ 0.5x + 0.5y &\leq 2 \\ x \geq 0, \quad y &\geq 0. \end{aligned}$$

To change to standard form, change the objective function and the first constraint.

5. Let x = number of kilograms of Super and
y = number of kilograms of Deluxe.

Maximize $z = 20x + 30y$
subject to

$$\begin{aligned} 0.5x + 0.25y &\leq 120 \\ 0.5x + 0.75y &\leq 160 \\ x \geq 0, \quad y &\geq 0. \end{aligned}$$

This model is in standard form.

7. Let x_1 = number of bags of Regular Lawn (in thousands)
x_2 = number of bags of Super Lawn (in thousands), and
x_3 = number of bags of Garden (in thousands).

Maximize $z = 300x_1 + 500x_2 + 400x_3$
subject to

$$\begin{aligned} 4x_1 + 4x_2 + 2x_3 &\leq 80 \\ 2x_1 + 3x_2 + 2x_3 &\leq 50 \\ x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 &\geq 0. \end{aligned}$$

This model is in standard form.

9. Let x_1 = number of books in paperback binding,
x_2 = number of books in bookclub binding, and
x_3 = number of books in library binding.

Maximize $z = 0.5x_1 + 0.8x_2 + 1.2x_3$
subject to

$$\begin{aligned} 2x_1 + 2x_2 + 3x_3 &\leq 420 \\ 4x_1 + 6x_2 + 10x_3 &\leq 600 \\ x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 &\geq 0. \end{aligned}$$

This model is in standard form.

11. Let x_{ij} = amount of the ith ingredient in the jth mixture (in kilograms), where

Ingredient 1 = Sunflower seeds
Ingredient 2 = Raisins
Ingredient 3 = Peanuts
Mixture 1 = Chewy
Mixture 2 = Crunchy
Mixture 3 = Nutty

$$\text{Maximize} \quad 2\sum_{i=1}^{3} x_{i1} + 1.6\sum_{i=1}^{3} x_{i2} + 1.2\sum_{i=1}^{3} x_{i3} - \sum_{j=1}^{3} x_{1j} - 1.5\sum_{j=1}^{3} x_{2j} - 0.8\sum_{j=1}^{3} x_{3j}$$

subject to

$$\begin{aligned}
\sum_{j=1}^{3} x_{1j} &\le 100 \\
\sum_{j=1}^{3} x_{2j} &\le 80 \\
\sum_{j=1}^{3} x_{3j} &\le 60 \\
0.6x_{11} - 0.4x_{21} + 0.6x_{31} &\le 0 \\
-0.2x_{11} - 0.2x_{21} + 0.8x_{31} &\le 0 \\
-0.4x_{12} + 0.6x_{22} + 0.6x_{32} &\le 0 \\
0.8x_{13} - 0.2x_{23} - 0.2x_{33} &\le 0 \\
0.6x_{13} + 0.6x_{23} - 0.4x_{33} &\le 0 \\
x_{ij} \ge 0, \quad i = 1,2,3; \quad j &= 1,2,3
\end{aligned}$$

Section 1.2, page 68

1. Maximize $z = \begin{bmatrix} 20 & 30 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$
   subject to

$$\begin{bmatrix} 0.4 & 0.3 \\ 0.2 & 0.4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \le \begin{bmatrix} 18 \\ 14 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} \ge \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

3. Maximize $z = 3x + 2y + 3v - 2w$
   subject to

$$\begin{bmatrix} 2 & 6 & 2 & -4 \\ -2 & -6 & -2 & 4 \\ 3 & 2 & -5 & 1 \\ -3 & -2 & 5 & -1 \\ 6 & 7 & 2 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ v \\ w \end{bmatrix} \le \begin{bmatrix} 7 \\ -7 \\ 8 \\ -8 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ v \\ w \end{bmatrix} \ge \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

5. Maximize $z = [-3 \quad -2 \quad 0 \quad 0]\begin{bmatrix} x \\ y \\ u \\ v \end{bmatrix}$

subject to

$$\begin{bmatrix} 2 & 1 & 1 & 0 \\ 3 & -2 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ u \\ v \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ u \\ v \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

7. Maximize $z = [3 \quad 2 \quad 3 \quad -2 \quad 0]\begin{bmatrix} x \\ y \\ v \\ w \\ u \end{bmatrix}$

subject to

$$\begin{bmatrix} 2 & 6 & 2 & -4 & 0 \\ 3 & 2 & -5 & 1 & 0 \\ 6 & 7 & 2 & 5 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ v \\ w \\ u \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ v \\ w \\ u \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

9. Maximize $z = [20 \quad 30 \quad 0 \quad 0]\begin{bmatrix} x \\ y \\ u \\ v \end{bmatrix}$

subject to

$$\begin{bmatrix} 0.5 & 0.25 & 1 & 0 \\ 0.5 & 0.75 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ u \\ v \end{bmatrix} = \begin{bmatrix} 120 \\ 160 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ u \\ v \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

11. (a) $\begin{bmatrix} 2 & 2 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 13 \end{bmatrix} \leq \begin{bmatrix} 8 \\ 15 \end{bmatrix}$; $\begin{bmatrix} 2 \\ 1 \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$; $z = 340$

$\begin{bmatrix} 2 & 2 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 8 \\ 14 \end{bmatrix} \leq \begin{bmatrix} 8 \\ 15 \end{bmatrix}$; $\begin{bmatrix} 1 \\ 3 \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$; $z = 420$

(b) $\begin{bmatrix} 2 & 2 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 8 \\ 18 \end{bmatrix} \leq \begin{bmatrix} 8 \\ 15 \end{bmatrix}$

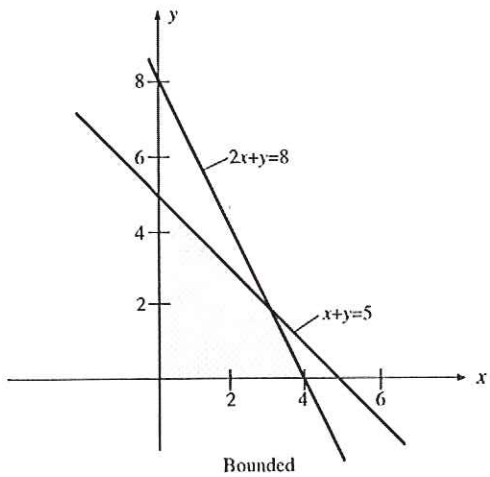
$\begin{bmatrix} 2 & 2 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 8 \\ 16 \end{bmatrix} \leq \begin{bmatrix} 8 \\ 15 \end{bmatrix}$

$\begin{bmatrix} -2 \\ 3 \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

13. (a) $x = 2, \quad y = 3, \quad u = 3, \quad v = 4$
(b) Impossible

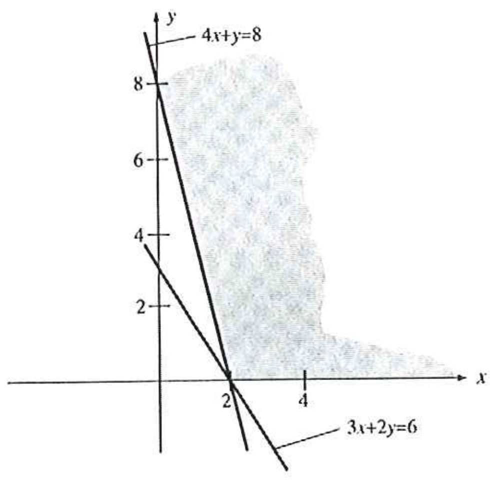
Section 1.3, page 81

1.



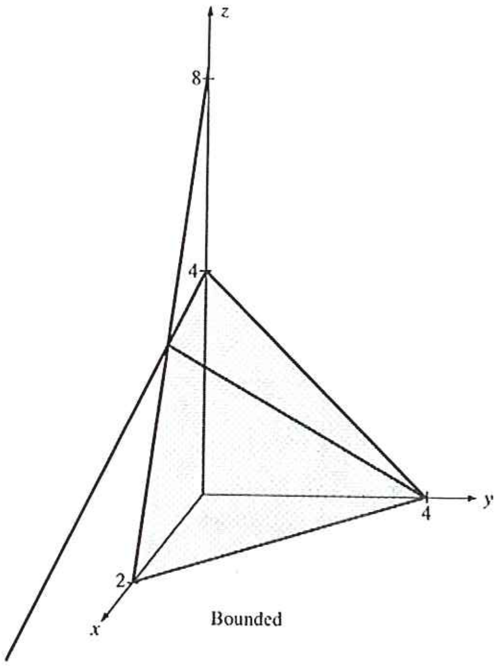
Bounded

3.



Unbounded

5.



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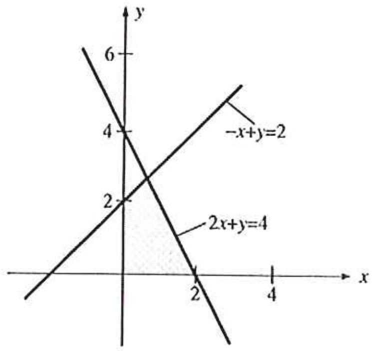

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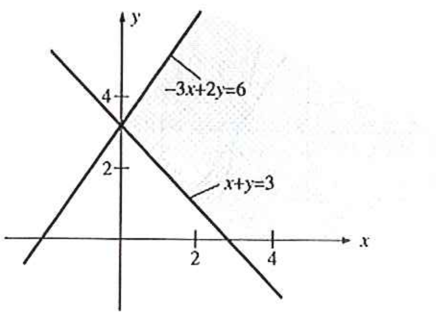
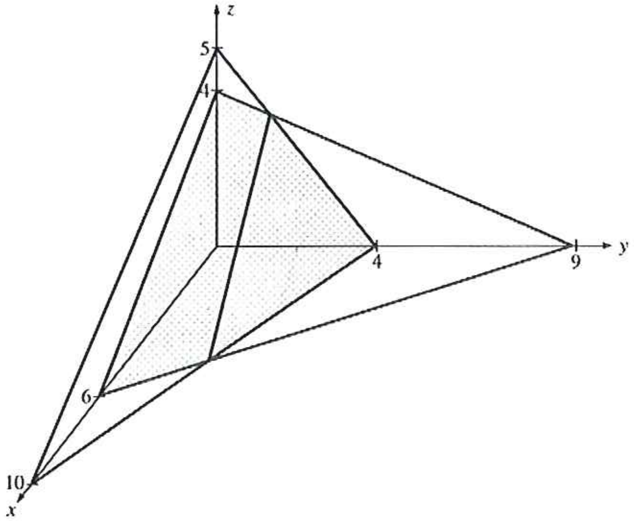
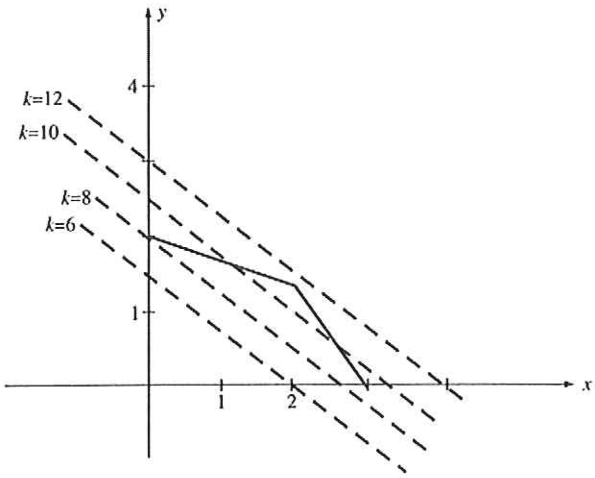


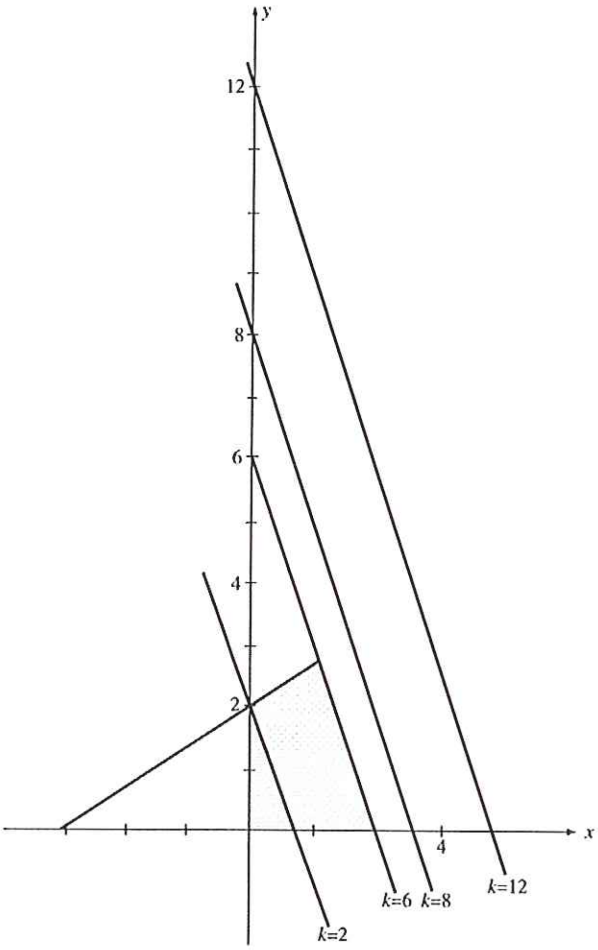
11.



13.



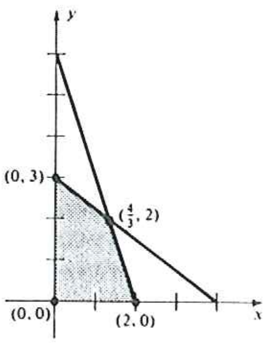
15.



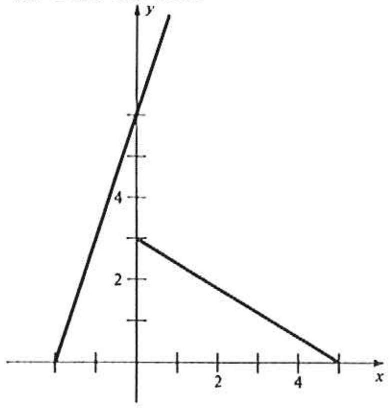
17. Not convex 19. Convex 21. Convex 23. Convex

Section 1.4, page 90

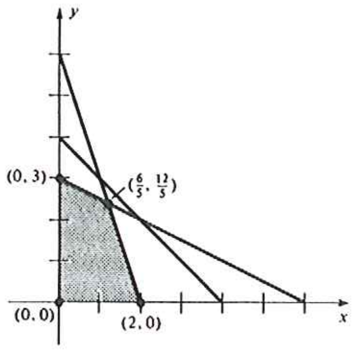
1. (a) (0, 0), (2, 0), ($\frac{4}{3}$, 2), (0, 3) (b) (0, 3); $z = 6$



3. (a) None exists (b) Does not exist

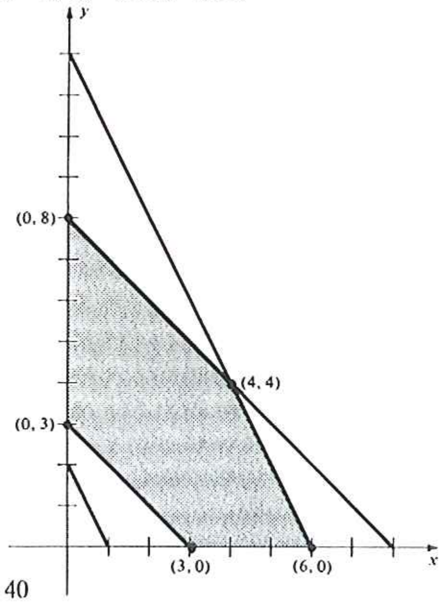


5. (a) (0, 0), (2, 0), $(\frac{6}{5}, \frac{12}{5})$, (0, 3)



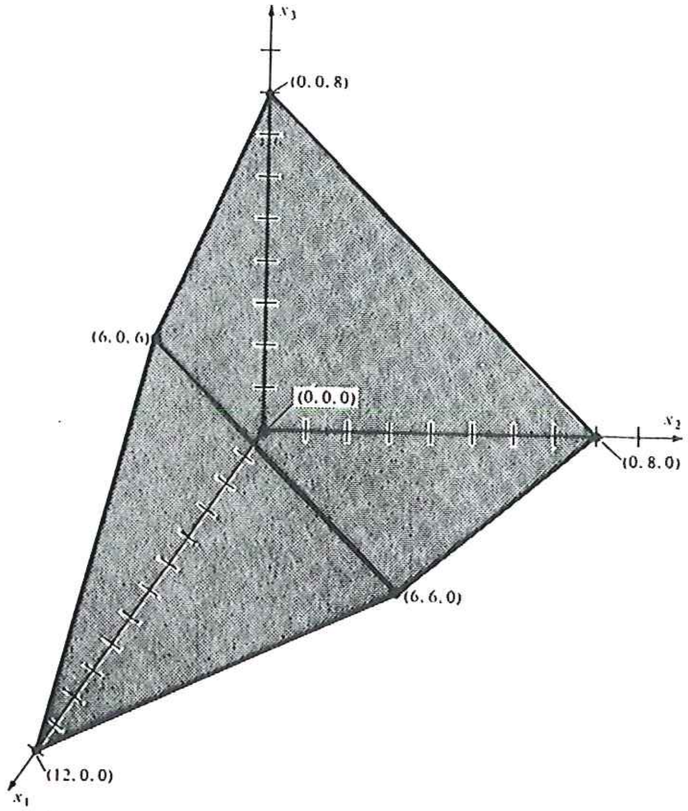
(b) (0, 0); $z = 0$

7. (a) (3, 0), (6, 0), (4, 4), (0, 8), (0, 3)



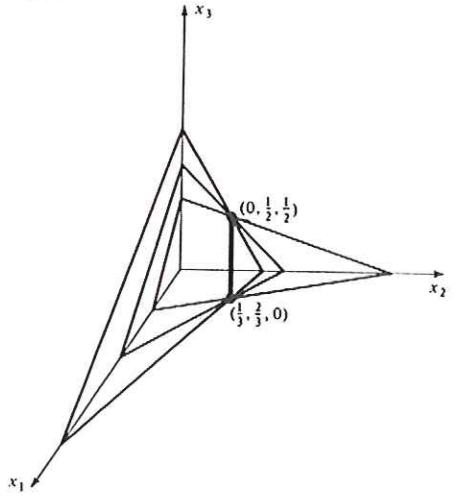
(b) (0, 8); $z = 40$

9. (a) (0, 0, 0), (0, 0, 8), (6, 0, 6), (12, 0, 0), (6, 6, 0), (0, 8, 0)



(b) (6, 6, 0); $z = 36$

11. (a) $(\frac{1}{3}, \frac{2}{3}, 0)$, $(0, \frac{1}{2}, \frac{1}{2})$



(b) $(\frac{1}{3}, \frac{2}{3}, 0)$; $z = 3$

Section 1.5, page 99

1. (i) (a), (c), (e) (ii) (a), (c) (iii) (a), (b), (c)
   (iv) (a) basic variables are x_2, x_4, x_5
   (c) basic variables are x_1, x_4, and one of x_2, x_3, x_5

3. Let x_1 = number of glazed doughnuts per day and
   x_2 = number of powdered doughnuts per day.

   Maximize $z = 0.07x_1 + 0.05x_2$
   subject to

$$\begin{aligned} x_1 + x_2 &\le 1400 \\ x_1 \quad\;\; &\le 1000 \\ x_2 &\le 1200 \\ x_1 \quad\;\; &\ge 600 \\ x_1 \ge 0, \quad x_2 &\ge 0 \end{aligned}$$

   Extreme points: (600, 0), (1000, 0), (1000, 400) (600, 800)
   Optimal solution: (1000, 400); $z = \$90$

5. Let x_1 = number of glazed doughnuts per day
   x_2 = number of powdered doughtnuts per day, and
   x_3, x_4, x_5, x_6 = slack variables.

   Maximize $z = 0.07x_1 + 0.05x_2$
   subject to

$$\begin{aligned} x_1 + x_2 + x_3 \qquad\qquad\qquad &= 1400 \\ x_1 \qquad\qquad + x_4 \qquad\qquad &= 1000 \\ + x_2 \qquad\qquad + x_5 \qquad &= 1200 \\ x_1 \qquad\qquad\qquad\qquad - x_6 &= 600 \end{aligned}$$

   At the optimal solution,

   $x_3 = 0$ = additional number of doughnuts per day that could be baked;
   $x_4 = 0$ = additional number of doughnuts per day that could be glazed;
   $x_5 = 800$ = additional number of doughnuts per day that could be dipped; and
   $x_6 = 400$ = number of glazed doughnuts over the required number.

   The basic variables are x_1, x_2, x_5, and x_6.

7. (a) Basic if x_2 and x_4 are taken as nonbasic variables; basic if x_1 and x_2 are taken as nonbasic variables; and not basic if x_1 and x_4 are taken as nonbasic variables
   (b) Not basic
   (c) Not basic

9. (a) Maximize $z = 4x_1 + 2x_2 + 7x_3$
subject to

$$\begin{aligned} 2x_1 - x_2 + 4x_3 + x_4 \quad &= 18 \\ 4x_1 + 2x_2 + 5x_3 \quad + x_5 &= 10 \\ x_j \geq 0, \quad j = 1, 2, \ldots, 5 \end{aligned}$$

(b)

x_1	x_2	x_3	x_4	x_5	Basic variables	Optimal
0	0	0	18	10	x_4, x_5	No
0	0	2	10	0	x_3, x_4	Yes
0	5	0	23	0	x_2, x_4	No
$\frac{5}{2}$	0	0	13	0	x_1, x_4	No

CHAPTER 2

Section 2.1, page 119

1.

	x	y	u	v	
u	3	5	1	0	8
v	2	7	0	1	12
	-2	-5	0	0	0

3. (a) x_2 (b) x_1 (c) No finite optimal solution

5. Using x_2 as the entering variable,

	x_1	x_2	x_3	x_4	
x_2	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{3}{2}$
x_3	$-\frac{1}{4}$	0	1	$-\frac{5}{4}$	$\frac{3}{4}$
	2	0	0	-2	$\frac{23}{2}$

Using x_4 as the entering variable,

	x_1	x_2	x_3	x_4	
x_4	1	2	0	1	3
x_3	1	$\frac{5}{2}$	1	0	$\frac{9}{2}$
	4	4	0	0	$\frac{35}{2}$

7.

	x_1	x_2	x_3	x_4	
x_1	1	1	5	0	4
x_4	0	1	7	1	10
	0	3	13	0	19

9. (a) $x_1 = 20, \quad x_2 = 0, \quad x_3 = 0, \quad u = 6, \quad v = 12, \quad w = 0$
Basic variables: x_1, u, v
(b)

	x_1	x_2	x_3	u	v	w	
x_1	1	0	−8	$-\frac{5}{2}$	0	13	5
x_2	0	1	2	$\frac{1}{2}$	0	−2	3
v	0	0	−5	−1	1	7	6
	0	0	7	$\frac{5}{2}$	0	−7	27

(c) $x_1 = 5, \quad x_2 = 3, \quad x_3 = 0, \quad u = 0, \quad v = 6, \quad w = 0$
Basic variables: x_1, x_2, v

11. $\begin{bmatrix} 0 \\ 0 \end{bmatrix}; \quad z = 0$

13. $x = \frac{38}{23}, \quad y^+ = \frac{12}{23}, \quad y^- = 0; \quad z = \frac{136}{23}$

15. Make 0 kg Super blend and $\frac{640}{3}$ kg Deluxe blend.
Profit = \$64.00

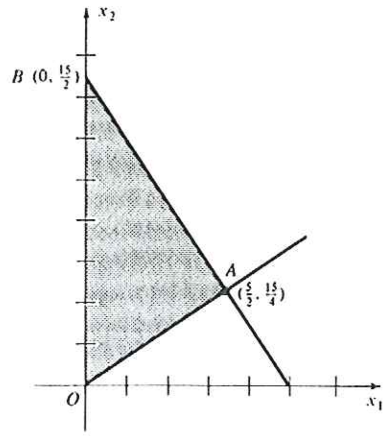
17. Make 0 books in either paperback or library bindings and 100 books in book-club binding. Profit = \$80.00

19. No finite optimal solution

21. $[3 \quad 0 \quad 0 \quad 0]^T; \quad z = 15$

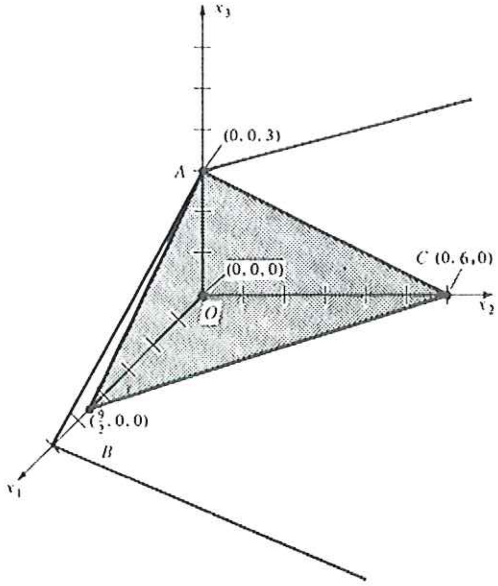
Section 2.2, page 130

1. $[0 \quad \frac{15}{2}]^T; \quad z = \frac{75}{2}$



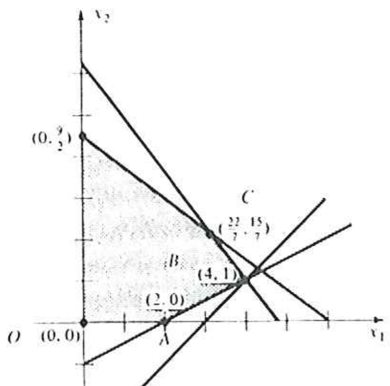
The simplex algorithm examines the following extreme points: O, O, A, B.

3. $[0 \quad 0 \quad 3]^T$; $z = 15$



The simplex algorithm examines the following extreme points: O, A, A.

5. $[\frac{22}{7} \quad \frac{15}{7}]^T$; $z = \frac{207}{7}$



The simplex algorithm examines the following extreme points: O, A, B, C.

9. $[4 \quad 1 \quad 0 \quad 4 \quad 1 \quad 0 \quad 0]^T$; $z = -2$

Section 2.3, page 150

1. (a)

	x_1	x_2	x_3	y_1	y_2	
y_1	1	2	7	1	0	4
y_2	1	3	1	0	1	5
	-2	-5	-8	0	0	-9

(b)

	x_1	x_2	x_3	y_1	y_2	
y_1	1	2	7	1	0	4
y_2	1	3	1	0	1	5
	$-1-2M$	$-5M$	$-3-8M$	0	0	$-9M$

3. (a)

	x_1	x_2	x_3	x_4	x_5	y_1	y_2	
y_1	1	3	2	-1	0	1	0	7
y_2	2	1	1	0	-1	0	1	4
	-3	-4	-3	1	1	0	0	-11

(b)

	x_1	x_2	x_3	x_4	x_5	y_1	y_2	
y_1	1	3	2	-1	0	1	0	7
y_2	2	1	1	0	-1	0	1	4
	$3-3M$	$-2-4M$	$-3M$	M	M	0	0	$-11M$

5.

	x_1	x_2	x_3	x_4	x_5	y_1	y_2	
x_2	$-\frac{1}{4}$	1	0	$-\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{4}$	$-\frac{3}{4}$	$\frac{1}{2}$
x_3	$\frac{3}{4}$	0	1	$-\frac{1}{4}$	$-\frac{1}{4}$	$\frac{1}{4}$	$-\frac{3}{4}$	$\frac{3}{2}$
	0	0	0	0	0	1	1	0