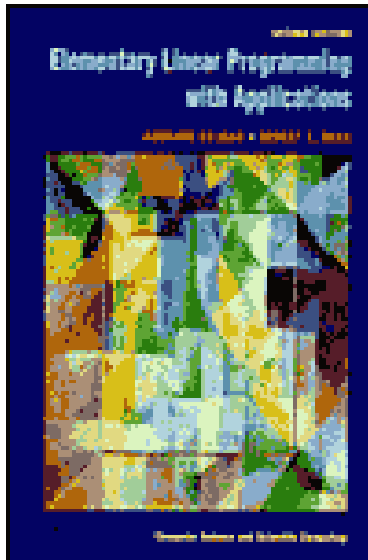


Elementary Linear Programming with Applications

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Preface

Classical optimization techniques have been widely used in engineering and the physical sciences for a long time. They arose from attempts to determine the “best” or “most desirable” solution to a problem. Toward the end of World War II, models for many problems in the management sciences were formulated and algorithms for their solutions were developed. In particular, the new areas of linear, integer, and nonlinear programming and network flows were developed. These new areas of applied mathematics have succeeded in saving billions of dollars by enabling the model builder to find optimal solutions to large and complex applied problems. Of course, the success of these modern optimization techniques for real problems is due primarily to the rapid development of computer capabilities in the past 40 years. Computational power has doubled every 12 months since 1964 (Moore’s Law, Joy’s Law) allowing the routine solution today of problems whose complexity was overwhelming even a few years ago.

With the increasing emphasis in mathematics on relevance to real-world problems, some of the areas of modern optimization mentioned above

have rapidly become part of the undergraduate curriculum for business, engineering, computer science, and mathematics students.

This book presents a survey of the basic ideas in linear programming and related areas and is designed for a one-semester or one-quarter course that can be taken by business, engineering, computer science, or mathematics majors. In their professional careers many of these students will work with real applied problems; they will have to formulate models for these problems and obtain understandable numerical answers. Our purpose is to provide such students with an impressive illustration of how simple mathematics can be used to solve difficult problems that arise in real situations and to give them some tools that will prove useful in their professional work.

A significant change that has taken place in the general teaching of this course has been the introduction of the personal computer. This edition takes due cognizance of this new development.

WHAT IS NEW IN THE SECOND EDITION

We have been very pleased by the widespread acceptance of the first edition of this book since its publication 15 years ago. Although many changes have been made in this edition, our objectives remain the same as in the first edition: to provide a textbook that is readable by the student, presents the basic notions of linear programming, and illustrates how this material is used to solve some very important problems that arise in our daily lives. To achieve these objectives we have made use of many faculty and student suggestions and have developed the following features for this edition.

FEATURES

- Some more review material on linear algebra has been added in Chapter 0.
- Chapters 1 and 2 of the first edition have been modified. In the revised Chapters 1 and 2, the material on the Extreme Point Theorem, basic solutions, and the Duality Theorem are now presented in separate sections. Moreover, the important elementary aspects of linear programming and its applications are covered more quickly and more directly.
- In Chapter 3, the presentation of the Duality Theorem has been rewritten, and now appears as Section 3.2.
- In Chapter 5, the presentations of the transportation problem, assignment problem, and maximal flow problem have been rewritten for greater clarity.

- New exercises have been added.
- New figures have been added.
- Throughout the book, the material on computer aspects has been updated.
- A computer disk containing the student-oriented linear programming code SMPX, written by Professor Evar D. Nering, Arizona State University, to be used for experimentation and discovery, is included with the book. Its use is described in Appendix C.
- Appendix A, new to this edition, provides a very elementary introduction to the basic ideas of the Karmarkar algorithm for solving linear programming problems.
- Appendix B has been added to this edition to provide a guide to some of the inexpensive linear programming software available for personal computers.

PRESENTATION

The Prologue gives a brief survey of operations research and discusses the different steps in solving an operations research problem. Although we assume that most readers have already had some exposure to linear algebra, Chapter 0 provides a quick review of the necessary linear algebra. The linear algebra requirements for this book are kept to a minimum. Chapter 1 introduces the linear programming problem, provides examples of such a problem, introduces matrix notation for this problem, and discusses the geometry of linear programming problems. Chapter 2 presents the simplex method for solving the linear programming problem. Chapter 3 covers further topics in linear programming, including duality theory and sensitivity analysis. Chapter 4 presents an introduction to integer programming, and Chapter 5 discusses a few of the more important topics in network flows.

The approach in this book is not a rigorous one, and proofs have been kept to a minimum. Each idea is motivated, discussed, and carefully illustrated with examples. The first edition of this book is based on a course developed by one of us (Bernard Kolman) under a College Science Improvement Program grant from the National Science Foundation.

EXERCISES

The exercises in this book are of three types. First, we give routine exercises designed to reinforce the mechanical aspects of the material under study. Second, we present realistic problems requiring the student to formulate a model and obtain solutions to it. Third, we offer projects,

some of which ask the student to familiarize himself or herself with those journals that publish papers in the subject under study. Most of the projects are realistic problems, and they will often have some vagueness in their statement; this vagueness must be resolved by the student as he or she formulates a model.

COMPUTERS

The majority of students taking this course will find that after having solved a few linear programming problems by hand, they will very much appreciate being able to use a computer program to solve such problems. The computer will reduce the computational effort required to solve linear programming problems and will make it possible to solve larger and more realistic problems. In this regard, the situation is different from when the first edition of this book appeared. Nowadays, there are inexpensive programs that will run on modest personal computers. A guide to some of these is provided in Appendix B. Moreover, bound with this book is a disk containing the program SMPX, developed by Evar D. Nering, Arizona State University, as courseware for a typical course in linear programming. This courseware allows the student to experiment with the simplex method and to discover the significance of algorithm choices.

Complementing SMPX courseware is LINDO, an inexpensive and powerful software package designed to solve linear programming problems. It was first developed in 1983 and is now available in both PC and Macintosh versions.

The final sections in each of Chapters 3, 4 and 5 discuss computer aspects of the material in the chapter. These sections provide an introduction to some of the features available in the linear programming codes used to solve large real problems and an introduction to the considerations that enter into the selection of a particular code.

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Prologue

Introduction to Operations Research

WHAT IS OPERATIONS RESEARCH?

Many definitions of *operations research* (frequently called OR) have been given. A common thread in these definitions is that OR is a *scientific method* for providing a quantitative basis for decision making that can be used in almost any field of endeavor. The techniques of OR give a logical and systematic way of formulating a problem so that the tools of mathematics can be applied to find a solution. However, OR differs from mathematics in the following sense. Most often mathematics problems can be clearly stated and have a specific answer. OR problems are frequently poorly posed: they arise when someone has the vague feeling that the established way of doing things can be improved. Engineering, which is also engaged in solving problems, frequently uses the methods of OR. A central problem in OR is the optimal allocation of scarce resources. In this context, scarce resources include raw materials, labor, capital, energy, and processing time. For example, a manufacturer could consult an operations research analyst to determine which combination of production techniques

should be used to meet market demands and minimize costs. In fact, the 1975 Nobel Prize in Economics was awarded to T. C. Koopmans and L. V. Kantorovich for their contributions to the theory of optimum allocation of resources.

DEVELOPMENT OF OPERATIONS RESEARCH

The use of scientific methods as an aid in decision making goes back a long time, but the discipline that is now called *operations research* had its birth during World War II. Great Britain, which was struggling for its very existence, gathered a number of its top scientists and mathematicians to study the problem of allocating the country's dwindling resources. The United States Air Force became interested in applying this new approach to the analysis of military operations and organized a research group. In 1947 George B. Dantzig, a member of this group, developed the simplex algorithm for solving linear programming problems. At approximately the same time the programmable digital computer was developed, giving a means of solving large-scale linear programming problems. The first solution of a linear programming problem on a computer occurred in 1952 on the National Bureau of Standards SEAC machine. The rapid development of mathematical programming techniques has paralleled the rapid growth of computing power. The ability to analyze large and complicated problems with operations research techniques has resulted in savings of *billions* of dollars to industry and government. It is remarkable that a newly developed discipline such as operations research has had such an impact on the science of decision making in such a short time.

PHASES OF AN OPERATIONS RESEARCH STUDY

We now look at the steps an operations analyst uses in determining information for decision making. In most cases the analyst is employed as a consultant, so that management has to first recognize the need for the study to be carried out. The consultant can now begin work using the following sequence of steps.

Step 1: Problem definition and formulation. In this phase the goal of the study is defined. The consultant's role at this point is one of helping management to clarify its objectives in undertaking the study. Once an acceptable statement of the goals has been made, the consultant must identify the decision alternatives. It is likely that there are some options that management will refuse to pursue; thus, the consultant will consider only the

alternatives acceptable to management. Attention must also be paid to the limitations, restrictions, and requirements of the various alternatives. For example, management might have to abide by fair employment laws or antipollution laws. Some alternatives may be limited by the available capital, labor, or technology.

- Step 2: **Model construction.*** The consultant now develops the appropriate mathematical description of the problem. The limitations, restrictions, and requirements must be translated into mathematical terms, which then give rise to the constraints of the problem. In many cases the goal of the study can be quantified as an expression that is to be maximized or minimized. The decision alternatives are represented by the variables in the problem. Often the mathematical model developed is one that has a familiar form and for which methods of solution are available.
- Step 3: **Solution of the model.*** The mathematical model developed in Step 2 must now be solved. The method of solution may be as simple as providing the input data for an available computer program or it may call for an investigation of an area of mathematics that so far has not been studied. There may be no method of finding a solution to the mathematical model. In this case the consultant may use heuristic methods or approximate methods, or it may be necessary to go back to Step 2 and modify the model. It should be noted that the solution to the model need not be the solution to the original problem. This will be further discussed below.
- Step 4: **Sensitivity analysis.*** Frequently the numbers that are given to the consultant are approximate. Obviously, the solution depends on the values that are specified for the model, and, because these are subject to variation, it is important to know how the solution will vary with the variation in the input data. For standard types of models these questions have been investigated, and techniques for determining the sensitivity of the solution to changes in the input data are available.
- Step 5: **Model evaluation.*** At this point the consultant has obtained a solution to the model, and often this solution will represent a solution to the given problem. The consultant must determine whether the answers produced by the model are realistic, acceptable to management, and capable of implementation by management. As in Step 1, the consultant now needs a thorough understanding of the nature of the client's business.
- Step 6: **Implementation of the study.*** Management must now decide how to implement the recommendations of the consultant.

Sometimes the choice is to ignore all recommendations and do something that is politically expedient instead.

THE STRUCTURE OF MATHEMATICAL MODELS

When a technical person discusses a model of a situation that is being studied, he or she is referring to some idealized representation of a real-life system. The model may simply involve a change in scale, such as the hobbyist's HO railroad or the architect's display of a newly planned community.

Engineers often use analogue models in which electrical properties substitute for mechanical properties. Usually the electrical analogues are much easier to deal with than the real objects. For example, resetting a dial will change the analogue of the mass of an object. Without the analogue one might have to saw off part of the object.

Mathematical models represent objects by symbols. The variables in the model represent the decision alternatives or items that can be varied in the real-life situation. There are two types of mathematical models: deterministic and probabilistic. Suppose the process described by the model is repeated many times. A *deterministic* model will always yield the same set of output values for a given set of input values, whereas a *probabilistic* model will typically yield many different sets of output values according to some probability distribution. In this book we will discuss only deterministic models.

The mathematical models that will be considered in this book are structured to include the following four basic components:

- (a) **Decision variables or unknowns.** Typically we are seeking values for these unknowns, which describe an optimal allocation of the scarce resources represented by the model. For example, decision variables might represent purchase lot size, number of hours to operate a machine, or which of several alternatives to choose.
- (b) **Parameters.** These are inputs that may or may not be adjustable by the analyst, but are known either exactly or approximately. For example, purchase price, rate of consumption, and amount of spoilage could all be parameters.
- (c) **Constraints.** These are conditions that limit the values that the decision variables can assume. For example, a variable measuring units of output cannot be negative; a variable measuring the amount to be stored cannot have a value greater than the available capacity.
- (d) **Objective function.** This expression measures the effectiveness of the system as a function of the decision variables. The decision

variables are to be determined so that the objective function will be optimized. It is sometimes difficult to determine a quantitative measure of the performance of a system. Consequently, several objective functions may be tried before choosing one that will reflect the goals of the client.

MATHEMATICAL TECHNIQUES IN OPERATIONS RESEARCH

The area of *mathematical programming* plays a prominent role in OR. It consists of a variety of techniques and algorithms for solving certain kinds of mathematical models. These models call for finding values of the decision variables that maximize or minimize the objective function subject to a system of inequality and equality constraints. Mathematical programming is divided into several areas depending on the nature of the constraints, the objective function, and the decision variables. Linear programming deals with those models in which the constraints and the objective function are linear expressions in the decision variables. Integer programming deals with the special linear programming situation in which the decision variables are constrained to take nonnegative integer values. In stochastic programming the parameters do not have fixed values but are described by probability distributions. In nonlinear programming some or all of the constraints and the objective function are nonlinear expressions in the decision variables. Special linear programming problems such as optimally assigning workers to jobs or optimally choosing routes for shipments between plants and warehouses have individually tailored algorithms for obtaining their solutions. These algorithms make use of the techniques of network flow analysis.

Special models, other than mathematical programming techniques, have been developed to handle several important OR problems. These include models for inventory analysis to determine how much of each item to keep on hand, for analysis of waiting-line situations such as checkout counters and tollbooths, and for competitive situations with conflicting goals such as those that would arise in a game.

Standard techniques for solving many of the usual models in OR are available. Some of these methods are iterative, obtaining a better solution at each successive iteration. Some will produce the optimal solution after a finite number of steps. Others converge only after an infinite number of steps and consequently must be truncated. Some models do not lend themselves to the standard approaches, and thus *heuristic* techniques—that is, techniques improvised for the particular problem and without firm mathematical basis—must be used.

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