1 <u>Sol. ex dati</u> 29/9/11 ok

<u>EX 1</u> • $D(f)$ $\quad f(x) = \sqrt{\log_2 (x^2-1)}$

$$\begin{cases} x^2 - 1 > 0, & |x| > 1 \\ \log_2 (x^2-1) \geq 0 & \rightarrow \quad x^2 - 1 \geq 1, \quad x^2 \geq 2, \quad |x| > \sqrt{2} \end{cases}$$

$\Rightarrow S = \{ |x| > \sqrt{2} \} =]-\infty, -\sqrt{2}[\cup]\sqrt{2}, +\infty[$

• $D(f)$, $\quad f = \dfrac{\sin \sqrt{x}}{3x - 1}$

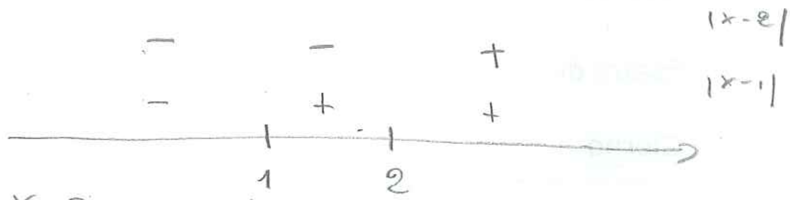
$$\begin{cases} x \geq 0 \\ 3x \neq 1, & x \neq \frac{1}{3} \end{cases}$$

$S = [0, \frac{1}{3}) \cup (\frac{1}{3}, +\infty)$

EX 2 • risolvere $|x-1| < |x-2|$ $\qquad$ <u>opp.</u> $|x-1|^2 < |x-2|^2$

$$|x-1| = \begin{cases} x-1, & x \geq 1 \\ -x+1 & x < 1 \end{cases}$$

$$|x-2| = \begin{cases} x-2 & x \geq 2 \\ -x+2 & < \end{cases}$$

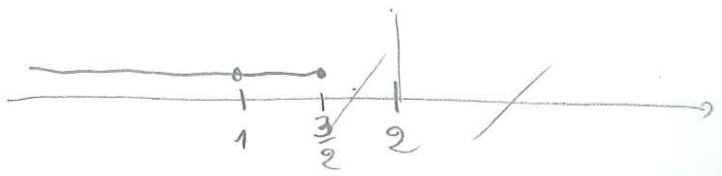


se $x < 1$ ho $\quad -x+1 < -x+2 \rightarrow 1 < 2$ sempre vero

se $x \in [1, 2[$ ho $\quad x - 1 < -x + 2 \rightarrow 2x < 3, \quad x < \frac{3}{2} = 1,5$

se $x \geq 2$ $\quad$ ho $x - 1 < x - 2 \rightarrow -1 < -2, \quad 1 > 2$ F

$\rightarrow S = (-\infty, \frac{3}{2})$



Da risolvere

$(1-\lg x)(x^2-3x+2)<0$

concordi:

$1-\lg x>0$ per $\lg x\leq 1$, $x<e$

$x^2-3x+2=(x-1)(x-2)>0$

valori esterni

$S = \cancel{(-\infty,1) \cup (2,e)}\ (1,2) \cup (e,+\infty)$

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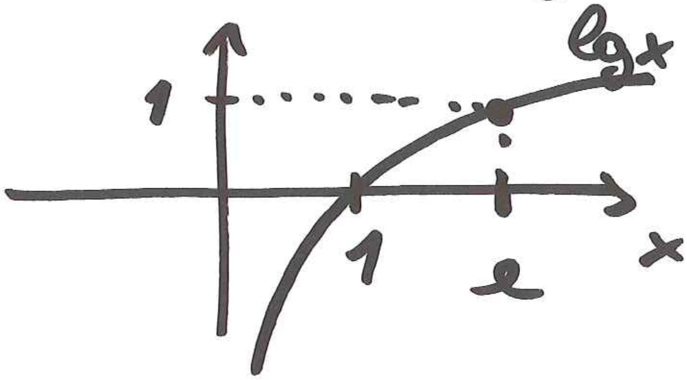
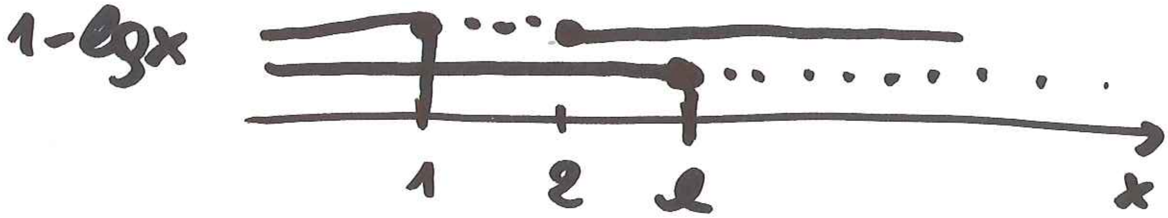
$$(1-\log x)\cdot(x^2-3x+2)<0$$

$$S=(1,2)\cup(e,+\infty)$$

$1-\lg x\geq 0$, $\lg x\leq 1=\log_e e$



$x\leq e$



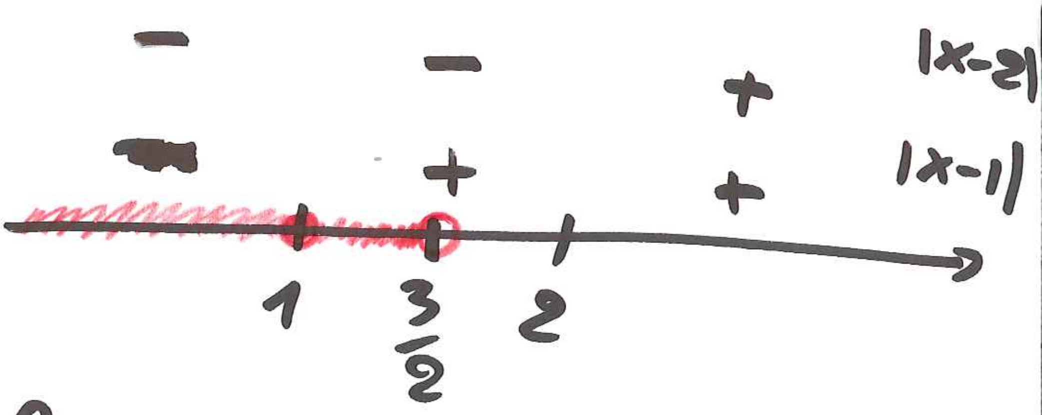
$x^2-3x+2\geq 0$ $\quad x_1=1,\ x_2=2$

EX $|x-1| < |x-2|$

$$S = \left(-\infty, \frac{3}{2}\right)$$

$$|x-1| = \begin{cases} x-1 & \text{se } x-1 \geq 0, \quad x \geq 1 \\ -x+1 & x < 1 \end{cases}$$

$$|x-2| = \begin{cases} x-2 & \text{se } x \geq 2 \\ -x+2 & x < 2 \end{cases}$$



$x < 1$: ho

$$-\cancel{x}+1 < -\cancel{x}+2 \quad \text{ok } \forall x$$

$x \in [1, 2[$ ho

$$x-1 < -x+2, \quad 2x < 3$$

$$x < \frac{3}{2}$$

2)

$x \geq 2$: ho

$\cancel{x}-1 < \cancel{x}-2$ F $\forall x$

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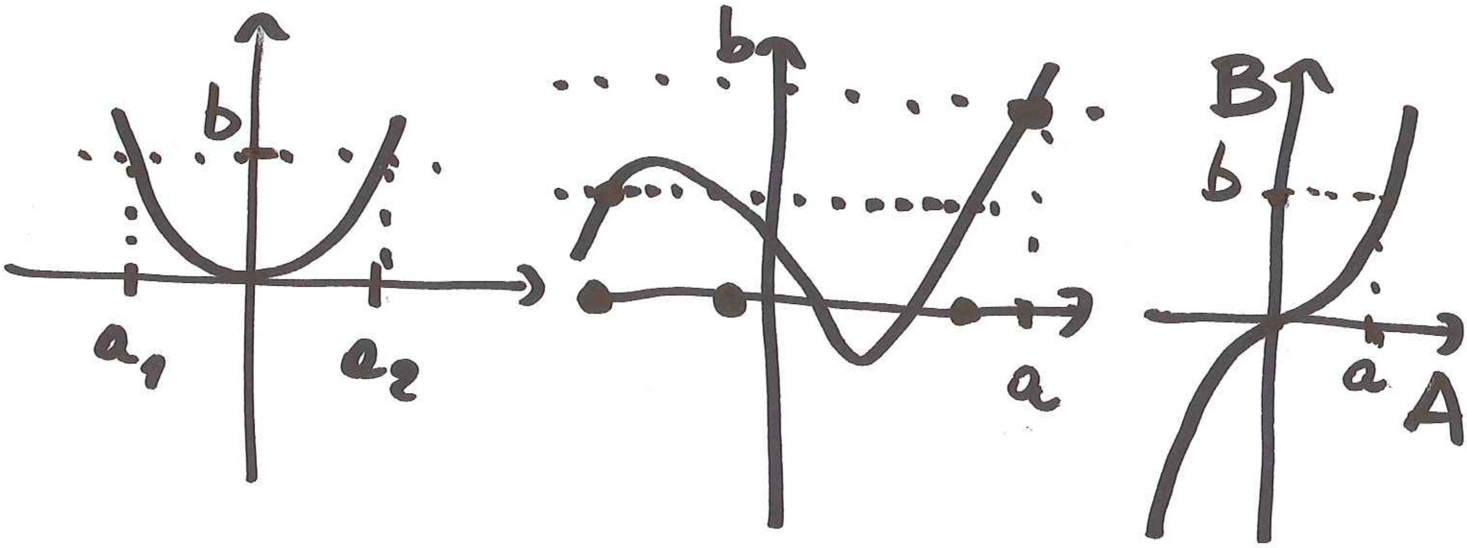
fz iniettiva



dato $b = y$, $f^{-1}(y) = a = x$

CONTROIMMAGINE

f iniett. <u>def</u>

$\forall\, b \in Cod(f)$ esiste un unico a controim. di b

$\forall\, b \in Cod(f)\ \exists!\ a \in D(f)$:

$$f^{-1}(b) = a$$

EX $g(x) = e^{2x+1}$

$g^{-1}(x)$?

dato y : $g^{-1}(y) = x$ cerco x

y è immag. di x

$$g(x) = y$$

$$e^{2x+1} = y$$

$$2x+1 = \lg y$$

$$x = \frac{\lg y - 1}{2} = g^{-1}(y)$$

$$g^{-1}(y) = \frac{\lg y - 1}{2}$$

$$g^{-1}(x) = \frac{\lg x - 1}{2}$$

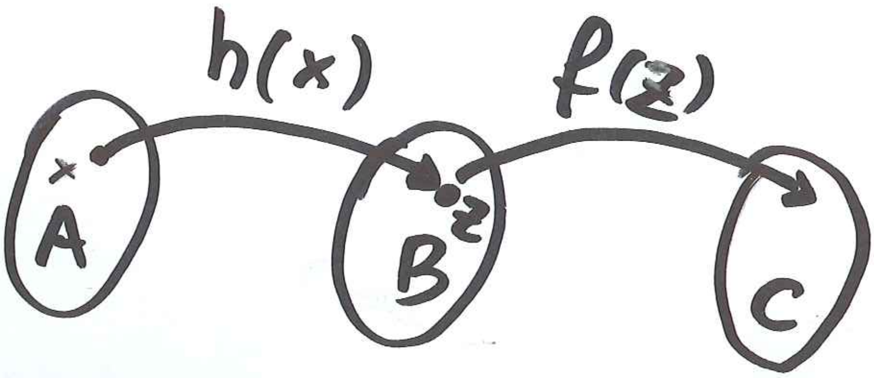
$$f(x) = 3 + \frac{1}{1+x^2}, \quad g(x) = e^{2x+1}$$

$$f(\underbrace{g^{-1}(x)}_{z}) = 3 + \frac{1}{1+z^2}$$

$$= 3 + \frac{1}{1+\left(g^{-1}(x)\right)^2} =$$

$$= 3 + \frac{1}{1+\left(\frac{\ln x - 1}{2}\right)^2}$$

$$f\left(g^{-1}(x)\right) = f(X) \Big|_{X \leftrightarrow g^{-1}(x)}$$



$$f(h(x)) = f(z)\Big|_{z=h(x)}$$

$$f \circ h : A \longrightarrow C$$

EX $\quad g(x) = x^2 + x$

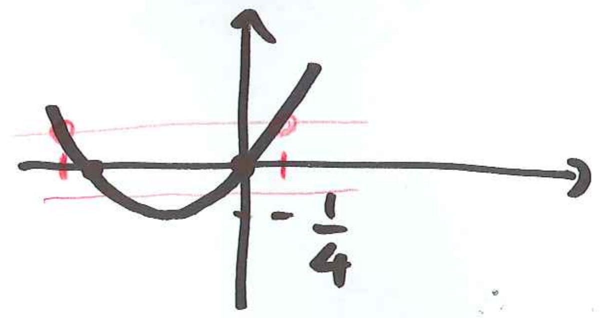
$y \in Cod(g): \quad g^{-1}(y)$

cerco x: $y = x^2 + x$, y parametro

$$x^2 + x - y = 0$$

$\Delta = 1 + 4y \geq 0$ se $4y \geq -1$, $\boxed{y \geq -\frac{1}{4}}$

$$Cod(g) = \left[-\frac{1}{4}, +\infty\right[$$



per $y \geq -\frac{1}{4}$. $x_{1,2} = \dfrac{-1 \pm \sqrt{\Delta}}{2}$

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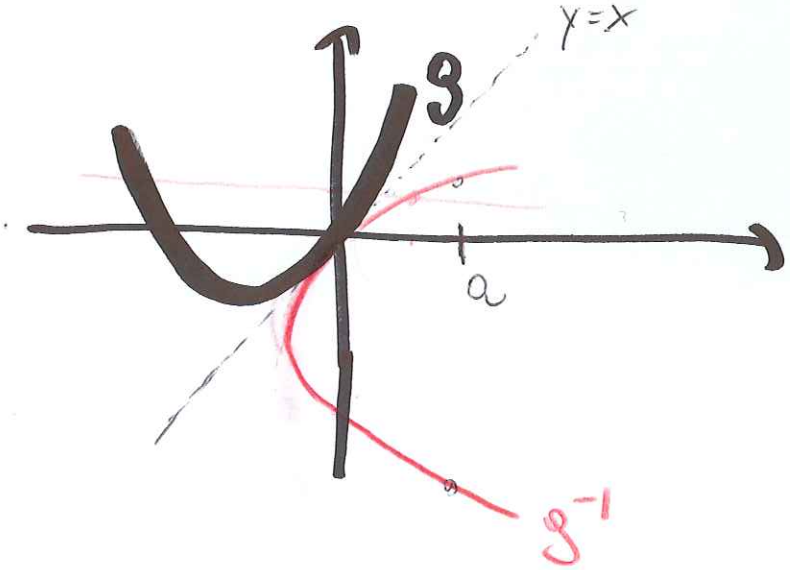
$$x_{1,2} = \frac{-1 \pm \sqrt{1+4y}}{2}$$

$$g^{-1}(y) = \left\{ \frac{-1+\sqrt{1+4y}}{2}, \frac{-1-\sqrt{}}{2} \right\}$$

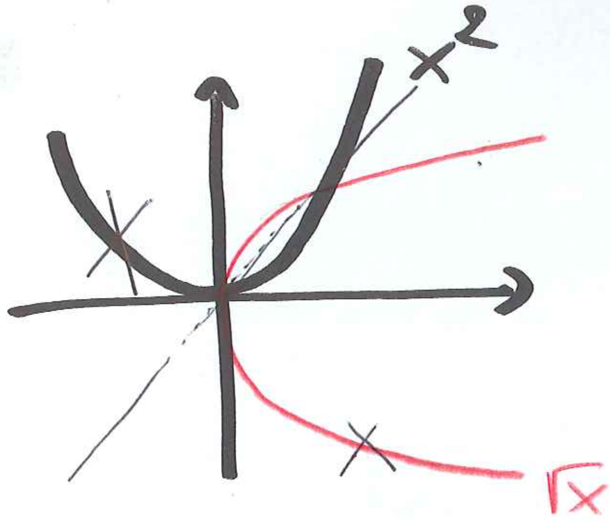
g^{-1} non è fz.

REGOLA GEN:

graf(f^{-1}) è simmetrico di graf(f) rispetto bisettr. I-III quadr.



ES. $g(x) = x^2$



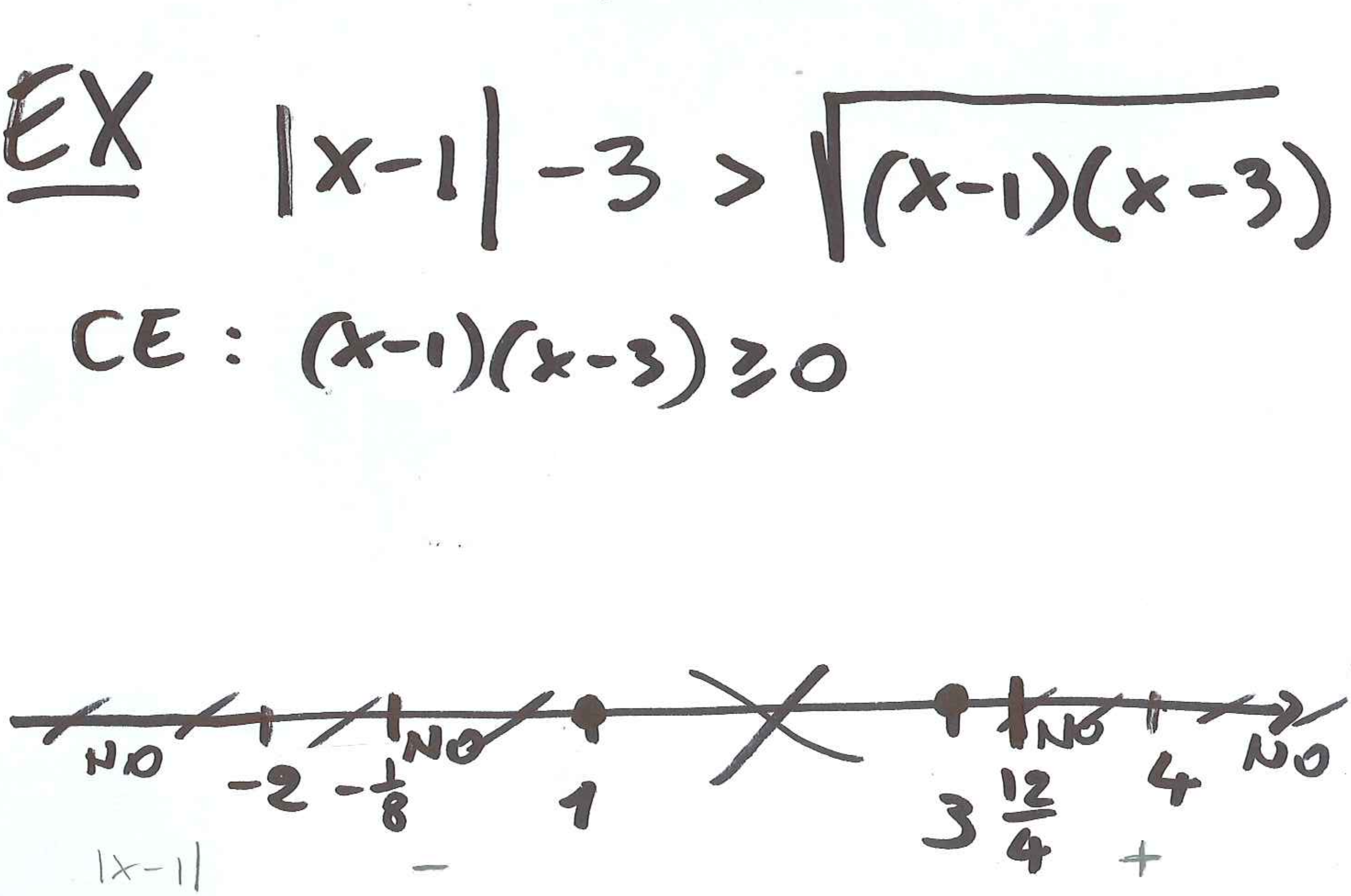
$\sqrt{x}$ radice aritmetica

$$\sqrt{(x-1)^2} \neq x-1$$

$$\sqrt{(x-1)^2} = |x-1|$$

EX $|x-1|-3 > \sqrt{(x-1)(x-3)}$

CE: $(x-1)(x-3) \geq 0$



se $x \leq 1$:

- $-x+1-3 > \sqrt{(x-1)(x-3)}$

$\underbrace{-x-2}_{a} > \underbrace{\sqrt{(x-1)(x-3)}}_{b}$

REGOLA GEN.

$a > b$ è equivalente ad $a^2 > b^2$

sse $a, b \geq 0$

se $-x-2 < 0$ no soluz.

$a<0,\ b>0:\ \underset{\text{mai}}{a > b}$

$x > -2$

se $x \leq -2$

$(-x-2)^2 > (x-1)(x-3)$

$x > -\frac{1}{8}$

$x \geq 3$:

$$x-1-3 > \sqrt{(x-1)(x-3)}$$

$$x-4 > \sqrt{(x-1)(x-3)}$$

se $x < 4$: DIS. IMPOSS.

se $x > 4$

$$(x-4)^2 > (x-1)(x-3)$$

$$x < \frac{13}{4}$$

no soluz $x > 4$

$$S = \emptyset$$

EX $|x-1| < |x-2|$ elevo al q.

OSS. $|a|^2 = a^2$

$$|-3|^2 = 3^2$$

$$|x-1|^2 < |x-2|^2$$

$$(x-1)^2 < (x-2)^2$$

$$x < \frac{3}{2}$$